Staple!

Ord: ___ ___

Linear Algebra
MAS4105 *6137*

Class-C

Prof. JLF King
Tuesday, 09Feb2016

C1: Show no work. Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$. Symbol B^{t} means the transpose of matrix B.

___ ___ **a** In $\mathbb{R}^3$, let $\mathbf{u} := (8, 5, -1)$, $\mathbf{v} := (1, 0, -4)$ and $\mathbf{w} := (13, 10, 10)$. Circle: Then $\mathbf{w} \in \mathrm{Line}(\mathbf{u}, \mathbf{v})$: *T* *F*

Then $\mathbf{w} \in \mathrm{Spn}(\mathbf{u}, \mathbf{v})$: *T* *F*

Then $\mathrm{Spn}(\mathbf{u}, \mathbf{v}, \mathbf{w}) = \mathbb{R}^3$: *T* *F*

___ ___ **b** VS $\mathbf{V} := \mathrm{MAT}_{4\times4}(\mathbb{R})$ is a 16-dim'al $\mathbb{R}$-VS. Define lin.trn $D\colon \mathbf{V} \to \mathbf{V}$ by $D(\mathsf{M}) := \mathsf{M} + \mathsf{M}^{\mathsf{t}}$. Then Nullity$(D) =$ ________. And Rank$(D) =$ ________.

The trn $U\colon \mathbf{V} \to \mathbf{V}$ by $U(\mathsf{M}) := \mathsf{M}^2$ is: Circle best

Linear   Affine (but not linear)   Not-affine

___ ___ **c** In each blank below, write either "there exist" or "for all", Circle one of the underlined scalar-pairs, and Circle a phrase.

Assertion $\mathrm{Spn}(\mathbf{v}, \mathbf{w}) \supset \mathrm{Spn}(\mathbf{x}, \mathbf{y})$ *means:*

" ________________ *scalars* $\underline{a, b}$ | $\underline{c, d}$ (*st.* | *we have that* | *and*)

________________ *scalars* $\underline{a, b}$ | $\underline{c, d}$ (*st.* | *we have that*)

$a\mathbf{v} + b\mathbf{w} = c\mathbf{x} + d\mathbf{y}$."

___ ___ **d** *Here*, let *AT* mean "Always True", *AF* mean "Always False" and *Nei* mean "Neither always true nor always false". Below, $\mathbf{v}, \mathbf{w}, \mathbf{x}$ repr. *distinct*, *non-zero* vectors in $\mathbb{R}^4$, a $\mathbb{R}$-VS. Please circle the correct response:

y1 If $\mathbf{x} \notin \mathrm{Spn}\{\mathbf{v}, \mathbf{w}\}$ then $\{\mathbf{v}, \mathbf{w}, \mathbf{x}\}$ is linearly independent. *AT* *AF* *Nei*

y2 Collection $\{\mathbf{0}, \mathbf{x}\}$ is linearly-independent.
AT *AF* *Nei*

y3 $\mathrm{Spn}\{\mathbf{v}, \mathbf{w}, \mathbf{x}, \mathbf{v} + 2\mathbf{w} + 3\mathbf{x}\}$ is all of $\mathbb{R}^4$.
AT *AF* *Nei*

y4 If none of $\mathbf{v}, \mathbf{w}, \mathbf{x}$ is a multiple of the other vectors, then $\{\mathbf{v}, \mathbf{w}, \mathbf{x}\}$ is linearly independent.
AT *AF* *Nei*

y5 For 2×2 matrices: $\mathrm{Det}(\mathsf{B} + \mathsf{A}) = \mathrm{Det}(\mathsf{B}) + \mathrm{Det}(\mathsf{A})$.
AT *AF* *Nei*

___ ___ **e** Let R_θ be the std. rotation [by θ] matrix. With

$$\mathsf{C} := \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix} \quad\text{and}\quad \mathsf{B} := \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix},$$

the product $[\mathsf{CB}]^{35} = \alpha \cdot \mathsf{R}_\theta$, with $\alpha =$ ________________ $\in \mathbb{R}_+$ and $\theta =$ ________________ $\in (-180°, 180°]$. [*Hint:* Don't multiply matrices. Geometrically, C and B represent what linear-trns?]

C3: OYOP: Essay: *Write on every third line, so that I can easily write between the lines.*

i On your essay-paper, write "*A* 5×7 *matrix* M *is in Reduced Row-Echelon Form IFF ...*" and complete the paragraph (with one or more sentences) to give a formal defn of RREF.

ii Give a careful proof of the...

1: RREF Uniqueness Theorem. *Consider two* 5×7 *RREF matrices* A *and* B. *If* A *is row-equivalent to* B, *then* A = B.◊

Start your argument with "Proof of the *RREF Uniqueness Thm*" and end it with "QED".

End of Class-C

C1:	___ ___ ___	145pts
C3:	___ ___	65pts
Total:	___ ___ ___	210pts

Please PRINT your name and ordinal. Ta:

Ord:

. . . .

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature:

. . . .