Staple!

Ord: ___ ___

Differential Eqns MAP2302 | **C-Class** | Prof. JLF King *Touch*: 17Oct2017

Bonjour. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with negative signs!)

Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797$\cdots$.

Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$.

C1: Show no work.

a $\underline{A}$ soln to $\big[f''-3f'\big](x)=14-6x$ is **polynomial** $f(x)=$ ⌊ . . . . . ⌋. Using parameters α and β, then, the *general* solution to $\big[h''-3h'\big](x)=14-6x$ is

$h_{\alpha,\beta}(x)=$ ⌊ . . . . . ⌋.

And $\underline{\text{the}}$ h with $h(0)=0$ and $h'(0)=0$ is $h(x)=$ ⌊ . . . . . ⌋.

b Consider linear DiffOp

$$\mathsf{V}(y) \;:=\; ty'' - [1+t]y' + y\,.$$

Verify [for yourself] that $\mathsf{V}(Y_0)=0$ and $\mathsf{V}(Y_1)=0$, where $Y_0:=\mathsf{e}^t$ and $Y_1:=1+t$. Their Wronskian is $\mathcal{W}(Y_0,Y_1)=$ ⌊ . . . . . ⌋. Then VoP tells us that $y_{\alpha,\beta}:=$ ⌊ . . . . . ⌋ is the general soln to $\mathsf{V}(y_{\alpha,\beta})=3t^2$.

c DiffOperators $\mathbf{P},\mathbf{Q},\mathbf{R},\mathbf{S}$ are defined as

$$\mathbf{P}(f) \;:=\; f(3)\cdot f'\,, \qquad \mathbf{Q}(f) \;:=\; \cos(3)\cdot f^{(3)}\,,$$
$$\mathbf{R}(f) \;:=\; [\cos(3)\cdot f] + f''\,, \qquad \mathbf{S}(f) \;:=\; \cos(3) + [3f']\,.$$

Then . . . **P** *is linear:* T F. **Q** *is linear:* T F.
R *is linear:* T F. **S** *is linear:* T F.

d We have $\big[6\cos(4t)-10\sin(4t)\big]\mathsf{e}^{7t} = \alpha\mathsf{e}^{Ut}+\beta\mathsf{e}^{\overline{U}t}$, for [possibly complex] numbers $\alpha=$ ⌊ . . . . . ⌋, $\beta=$ ⌊ . . . . . ⌋, $U=$ ⌊ . . . . . ⌋.

OYOP: *In grammatical English* ***sentences****, write your essay on every* ***third*** *line* (usually)*, so that I can easily write between the lines.*

C2: Brine with $5\frac{\mathsf{lb}}{\mathsf{gal}}$ salt flows at rate $1\frac{\mathsf{gal}}{\mathsf{min}}$ *into* a tank that initially held 100gal of $2\frac{\mathsf{lb}}{\mathsf{gal}}$-salt brine. The tank is well-mixed, and brine is flowing *out* at rate $3\frac{\mathsf{gal}}{\mathsf{min}}$. So the tank will empty in ⌊ . . . . . ⌋ minutes.

At time t, let $\sigma(t)$ denote the tank-salinity [in lb/gal] and use $y(t)$ for the total number of pounds of salt in the tank.

Explain how to derive a DE for $y()$. Don't just pull a DE out of the air; **explain**, using *Text* and *Pictures*, $\underline{\text{how}}$ it comes from the physical situation. Now re-write the DE in linear-DE form. Use FOLDE to solve the DE. Dividing by the amount of water in the tank at time t, gives this formula for the salinity:

$\sigma(t)=$ ⌊ . . . . . ⌋ $\frac{\mathsf{lb}}{\mathsf{gal}}$.

As the tank approaches empty, its salinity approaches ⌊ . . . . . ⌋ $\frac{\mathsf{lb}}{\mathsf{gal}}$. *Do Not approximate.*

End of C-Class

C1:	___ ___ ___	105pts
C2:	___ ___	80pts
Total:	___ ___ ___	185pts

Please PRINT your name and ordinal. Ta:

Ord: ⌊ . . . . . ⌋

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ⌊ . . . . . ⌋