



Staple!

NT-Cryptography  
MAT4930 2H22

Class-C

Prof. JLF King  
Wedn., 10Apr2019

Please fill-in every *blank* on this sheet.

.....

**C1:** Show no work. Write DNE if the object does not exist or the operation cannot be performed.  $\mathcal{N}B: DNE \neq \{\} \neq 0 \neq$  Empty-word.

**a** A minimum requirement for an LOR (letter-of-recommendation) from Prof. K is two courses.  Circle:

Yes

True

Darn tootin'!

**b** Entropy  $\mathcal{H}(\frac{1}{16}, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}) =$  .....

**c** Dictionary is 1: $\epsilon$ , 2:'0', 3:'00', 4:'1'. Thus  $\text{EnZiv}(0010000011111001) =$  .....

..... in  $\langle 7 \rangle 1 \langle 4 \rangle 0 \dots$   noise notation. In bits sent through the channel,  $\text{EnZiv}(0010000011)$  is .....

Ord: \_\_\_\_\_

**d**

Evaluate  $Id \otimes \mu$  on the numbers 1, 2, 5, 10, 20:

Mystery function  $f: \mathbb{Z}_+ \rightarrow \mathbb{Z}$  satisfies  $f \otimes \tau = Id$ . So  $f(20) =$  .....

OYOP: In grammatical English **sentences**, write your essay on every 2<sup>nd</sup> line (usually), so that I can easily write between the lines.

**C2:** State *Shannon source-coding thm* as stated/proved in class.

**C3:** State *Jensen's Inequality* as a formal thm, including the IFF condition, and *defining* [not just naming] what kind of fnc  $f: \mathbb{R} \rightarrow \mathbb{R}$  that *Jensen's* applies to.

Draw a LARGE, labeled picture illustrating the idea of the proof, but do not write a proof.

**C4:** Let  $\vec{1} := 1111\dots$ , the half- $\infty$  constant-1 bit-string. Using our Ziv-algorithm, with dictionary that [initially] only has the nullword, we start parsing  $\vec{1}$ .

Let  $P(k)$  be the largest-number of bits we've parsed, having used-up at most  $k$  many bits from  $\vec{1}$ . I.e, we Ziv-parse, and we eventually parse a new word [which we enter into our dictionary], having read exactly  $P(k)$  many bits, in total, where  $P(k) \leq k$ . As we scan for the next new word, we run past the  $k^{\text{th}}$ -bit in  $\vec{1}$ .

**i** Give an approximate formula for  $N(k)$ , the number of words you've parsed, having read the first  $P(k)$  many bits.

**ii** Let  $Z(k)$  be the length of the Ziv-compressed bit-string that encodes the first  $P(k)$  many bits in the source-string. When  $k$  is large, give a pretty good estimate for  $Z(k)$ ; a “closed formula”, neither having a  $\sum$  summation operator, nor a  $\prod$  product operator.

What are approximate values for  $N(500,000)$ , and for  $Z(500,000)$ ?

Compute  $\lim_{k \rightarrow \infty} \frac{Z(k)}{k} = \text{.....}$ .

**C1:** \_\_\_\_\_ 105pts

**C2:** \_\_\_\_\_ 45pts

**C3:** \_\_\_\_\_ 50pts

**C4:** \_\_\_\_\_ 85pts

**Total:** \_\_\_\_\_ 285pts

**HONOR CODE:** *“I have neither requested nor received help on this exam other than from my professor (or his colleague).”*  
*Name/Signature/Ord*

Ord: \_\_\_\_\_