Advanced Calc
MAA4102 & MAA5104

Class-C

Prof. JLF King
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Abbrevs. **seq** for "sequence". Use **nv-** for "non-void", e.g "a nv-closed set K". Use **MS** for "metric space".

Use $\mathbb{I}$ for the set $\mathbb{R} \setminus \mathbb{Q}$ of irrationals. Use $\overline{\mathbb{R}}$ for $[-\infty, +\infty]$, the "extended reals". Use **LUBP** for "(the) LUB-property". Use **GLBP** for "(the) greatest lower bound property".

For each of the limit questions, write "$+\infty$", "$-\infty$", a real number, or –*if none of these*– "DNE". Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. For the **True**/**False** questions, circle *T* or *F*, **but** *write in* "*DNE*" if the question is inconsistent.

C1: Show no work.

a. Ordered set $(\mathbb{I}, <)$ has LUBP. *T* *F*

$\{K + \frac{1}{n} \mid K, n \text{ posints}\}$ has LUBP. *T* *F*

Vogon poetry has the LUB-property. *T* *F*

b. Fnc $h(x) := x + \cos(x)$ is unif-cts on $\mathbb{R}$. *T* *F*

Fnc $x \mapsto \mathsf{e}^x$ is unif-cts on interval $(-\infty, 0]$. *T* *F*

c. Spoze $q \in \mathbb{Q} \setminus \{0\}$ and $\alpha, \beta \in \mathbb{I}$.

Then "$q \cdot \alpha \in \mathbb{I}$": *AT* *AF* *Nei*

And "$\beta \cdot \alpha \in \mathbb{I}$": *AT* *AF* *Nei*

d. Floor/ceil: $\lfloor \frac{780}{7} \rfloor \cdot 7 =$ _____. $\lceil \frac{780}{7} \rceil \cdot 7 =$ _____.

For all x real: $\lfloor -x \rfloor = -\lceil x \rceil$. *T* *F*

e. $\frac{\mathrm{d}}{\mathrm{d}x}(2^x) =$ _____. Let $F(x) := 2^{3^x}$. Its derivative, then, is $F'(x) =$ _____.

f. On $\mathbb{R}$, say $x \$ y$ IFF $x - y \in \mathbb{Z}$. Then $\$$ is:

Transitive: *T* *F*. **Symmetric**: *T* *F*.

Reflexive: *T* *F*.

For two fncs f, g:$\mathbb{R}\circlearrowright$, say $f \bowtie g$ IFF **there exists** $x \in \mathbb{R}$ with $f(x) = g(x)$. Then $\bowtie$ is:

Trans.: *T* *F*. **Symm.**: *T* *F*. **Reflex.**: *T* *F*.

g. These *open* bnded nv-intervals $U_n :=$ _____, when intersected, form a set $\bigcap_{n=1}^{\infty} U_n =$ _____ which is neither open (*partial credit*) nor closed.

Essays. On your own sheets of lined paper, give the following definitions or proofs. No "scratch work" accepted, only complete, grammatical, coherent sentences. Write **every 2nd or every 3rd line** for math essays.

C2: On a set Ω, a fnc $\mathsf{m}:\Omega\times\Omega\to[0,\infty)$ is a ***metric*** if: $\forall P, Q, R \in \Omega$: (*Write the 3 remaining axioms.*)

C3: On MS (Ω, m), *define* the ball $\mathrm{Ball}_r(P)$, where $P \in \Omega$ and $r > 0$. Now, given two Ω-open sets U and V, carefully prove that $U \cap V$ is open.

C4: i. A sequence $\vec{\mathbf{b}} \subset \mathbb{R}$ is ***Cauchy*** IFF . . . [*Hint:* Be precise with your quantifiers]

ii. Write the *definition* of "$\lim_{x\to 4} \log(x) = 5$". [*Hint:* You need 3 quantifiers, an ε and a δ.]

iii. Given seqs. with $\lim(\vec{\mathbf{a}}) = 2$ and $\lim(\vec{\mathbf{c}}) = 4$, let $\vec{\mathbf{p}} := \vec{\mathbf{a}} \cdot \vec{\mathbf{c}}$. Given $\varepsilon > 0$, produce –with careful proof– a posint N st.: $\forall k \geqslant N$: $|p_k - 8| < \varepsilon$.

C1:	___ ___ ___	175pts
C2:	___ ___	45pts
C3:	___ ___	75pts
C4:	___ ___	95pts
KnowsOrd:	___	5pts
Total:	___ ___ ___	395pts

Print name _____ Ord: _____

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: _____