Linear Algebra
MAS3114 3164

Prac-C

Prof. JLF King
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Hello. Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797⋯.

C1: Show no work.

a State the Cramer's Rule Thm.

Apply C.R. to give a formula for $x_2 =$ ______

ITOf A, B, C, D, t_1, t_2, where $\begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$.

b $\mu=$ ______ $\leqslant \nu =$ ______

are the eigenvalues of $\mathsf{G} := \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$. Let $\mathsf{D} := \begin{bmatrix} \mu & 0 \\ 0 & \nu \end{bmatrix}$. Then $\mathsf{D} = \mathsf{U}^{-1}\mathsf{G}\mathsf{U}$ where the 2×2 <u>*integer*</u> matrix U is

$$\mathsf{U} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}.$$

c The seq. $\vec{\mathbf{g}} := (g_n)_{n=-\infty}^{\infty}$ is defined by recurrence

$$g_{n+2} = 3g_{n+1} + 4g_n$$

and initial conditions $g_0 := 2$ and $g_1 := 1$. So its n^{th} term is $g_n = C_1 \cdot \mu^n + C_2 \cdot \nu^n$, where $\mu < \nu$ are real, and

$C_1=$ ______, $\mu=$ ______,

$C_2=$ ______ and $\nu=$ ______.

[*Hint:* The corresponding matrix is $\mathsf{G} := \begin{bmatrix} 3 & 4 \\ 1 & 0 \end{bmatrix}$. And μ, ν are its eigenvalues.]

d $\mathsf{M} := \begin{bmatrix} -5 & 3 & 18 \\ -2 & 0 & 12 \\ -4 & 4 & 7 \end{bmatrix}$ has three real eigenvalues,

$\alpha=$ ______ $\leqslant \beta=$ ______ $\leqslant \gamma=$ ______.

Hence $\begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} = \mathsf{U}^{-1}\mathsf{M}\mathsf{U}$, where

$$\mathsf{U} = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix}.$$

e An example of 2×2-matrices with $\mathsf{A}^2 \neq \mathsf{B}^2$, yet with $\mathsf{A}^3 = \mathsf{B}^3$, is $\mathsf{A} =$ ______ and $\mathsf{B} =$ ______.

f The 3×3 elem-matrix whose lefthand action adds 8 times row-2 to row-1 is

$$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}.$$

g Suppose T is a linear map from $\mathbb{R}^3$ to $\mathbb{R}^2$. Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis for $\mathbb{R}^3$, and let $\{\mathbf{v}_1, \mathbf{v}_2\}$ be the standard basis for $\mathbb{R}^2$. Suppose that $T(\mathbf{e}_1) = 17\mathbf{v}_1 - 2\mathbf{v}_2$ and $T(\mathbf{e}_2) = 6\mathbf{v}_2$ and $T(\mathbf{e}_3) = -4\mathbf{v}_1 - 3\mathbf{v}_2$.

Then the matrix of T is: ______.

h The matrix-product $\begin{bmatrix} 2 & 1 & 4 \\ 0 & -1 & 1 \\ -2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{2} & 0 & \sqrt{2} \\ 0 & -1 & 5 \end{bmatrix}$ equals

______

i Matrix-product $\begin{bmatrix} b \\ c \end{bmatrix} \cdot \begin{bmatrix} x & y \end{bmatrix} =$ ______.

j Over $\mathbb{Q}$, the inverse of $\mathsf{E} := \begin{bmatrix} 1 & x & z \\ & 1 & y \\ & & 1 \end{bmatrix}$ is

$$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}.$$

k The inverse of $\begin{bmatrix} 1 & -6 & -7 \\ & -3 & \\ & & 1 \end{bmatrix}$ is $\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}$.

l Let $\mathsf{K} := \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$. The minimum-deg <u>**monic**</u> polynomial $h()$ st. $h(\mathsf{K}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is $h(x)=$ ______.

m Consider these two matrices:

$$\mathsf{R} := \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \quad \text{and} \quad \mathsf{A} := \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}.$$

Product matrix $[RA]^{40}$ equals $\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$.

[*Hint:* You don't need multiply matrices. Geometrically, what motion do these matrices represent.]

n Consider these two matrices:

$$\mathsf{C} := \tfrac{1}{2} \cdot \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix} \quad \text{and} \quad \mathsf{B} := \tfrac{1}{2} \cdot \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} \end{bmatrix}.$$

Determine the matrix $[\mathsf{CB}]^{44}$= ________.

[*Hint:* You don't need multiply matrices. Geometrically, what motion do these matrices represent.]

o Let $\mathbf{v}_1 := \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 := \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$, $\mathbf{v}_3 := \begin{bmatrix} 4 \\ Y \\ 3 \end{bmatrix}$. Our $\mathbf{v}_3$ is in $\text{Spn}(\mathbf{v}_1, \mathbf{v}_2)$ when number Y= ________. And then, $\mathbf{v}_3 = \alpha\mathbf{v}_1 + \beta\mathbf{v}_2$, where α= ________ and β= ________.

p Let $\mathbf{v}_1 := \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 := \begin{bmatrix} 2 \\ 1 \\ -2 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 := \begin{bmatrix} 4 \\ W \\ 3 \\ Y \end{bmatrix}$, So $\mathbf{v}_3 \in \text{Spn}(\mathbf{v}_1, \mathbf{v}_2)$ when W= ________ & Y= ________. And $\mathbf{v}_3 = \alpha\mathbf{v}_1 + \beta\mathbf{v}_2$, where α= ________ and β= ________.

Essay question: On your own sheets of paper, write a soln using complete sentences, explaining a bit about HOW this problem is solved.

C2: A system of 3 linear equations in unknowns $x_1, \dots, x_5$ reduces to the augmented matrix

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 1 & 12 \\ 0 & 0 & 1 & 0 & -8 & 34 \\ 0 & 0 & 0 & 1 & 5 & -56 \end{array}\right],$$

which is *almost* in RREF. Please circle each *pivot*.

OYOP, describe the *general solution* in this form,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \alpha \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \beta \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \gamma \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \dots$$

where each $\alpha, \beta, \gamma, \delta, \dots$ is a free variable (either x_1 or...or x_5), and each column vector has specific numbers in it. $\text{Dim}(\textit{SolnFlat})$ = ________.