Abstract Algebra
MAS4301 3175 **Prac-Cl-C** Prof. JLF King
Monday, 31Mar2008

C3: Show no work.

a   A finite group G acts on a finite set Ω. Then the number of G-orbits divides $\mathrm{Ord}(G)$: **T** **F**

b   Let $V_K := G \times \overset{K}{...} \times G$, where $G := (\mathbb{Z}_3, +)$. AAD-POIntegers, $|\mathrm{Aut}(V_3)| =$ _____.

c K-color the *edges* of a tetrahedron. There are

_____ rotation-distinct

colorings. [*Hint:* Use Burnside's Thm, P.490.]

d   There exist groups P, A, B with $P \times A \cong P \times B$, yet $A \not\cong B$. $\boxed{Circle}$ one: **True** **False**

e   In **CN**, $\alpha^4 = (0\,8\,7\,3\,5\,2\,6\,4\,1) \in \mathbb{S}_7$. So $\alpha = $ _____

f $G := \mathbb{Z}_{625} \times \mathbb{Z}_{125} \times \mathbb{Z}_7$ has _____ *elements* of

order-25. So G has _____ *cyclic subgps* of order-25.

   Essay questions: **Fill-in all blanks.** *For each question, carefully write a triple–spaced essay solving the problem.*

C4: For subgps $H, K \subset G$, suppose that $K \lhd G$. Prove that the set-product HK is a sub*group* of G. [*Hint:* First show that $KH = HK$.]

C5: **a**   Let $H := \mathrm{Z}(G)$. Prove that $H \lhd G$. More strongly, prove that H is a *characteristic subgp* of G; that $H \overset{\mathrm{Aut}}{\lhd} G$.

b   Suppose $f{:}G{\to}\Gamma$ is a gp-homomorphism. Give the *definition* of $K := \mathrm{Ker}(f)$.   Prove that $K \lhd G$.
   Give an example of *specific groups* G, Γ and a homomorphism $f{:}G{\to}\Gamma$ for which $\mathrm{Ker}(f)$ is *not* a characteristic subgp of G.

c   For an elt $h \in G$, the inner-aut $J_h(x) := hxh^{-1}$. For an $\alpha \in \mathrm{Aut}(G)$, prove that $\boxed{\alpha \circ J_h \circ \alpha^{-1} = J_{\alpha(h)}}$. Use this to argue that $\mathrm{Inn}(G) \lhd \mathrm{Aut}(G)$.

C6:   Recall that ⊓⊓-auts $S := \begin{smallmatrix} \times & \times \\ \times & \times \end{smallmatrix}$ and

$T := |\!\times\!|$. Draw LARGE 16-dot diagrams of $T\mathsf{S}$ and $\beta :=$

$T\mathsf{SR}$. Cell-cycle-sig(β) is

_____

and ⊓⊓-cycle-sig$(\beta) =$ _____ .

C-Home:	__ __ __	265pts
C3:	__ __	95pts
C4:	__ __	15pts
C5:	__ __	20pts
C6:	__ __	20pts
Total:	__ __ __	415pts