Differential Eqns
MAP2302

Some problems

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Here some some problems. This is neither indicative of the length nor the format of the exam. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with negative signs!)

Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797$\cdots$.

Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$.

P1: Show no work.

a DE $\left[\mathcal{N}(x,y)\cdot\frac{\mathrm{d}y}{\mathrm{d}x}\right]+\mathcal{M}(x,y)=0$ is *exact*, where

$$\mathcal{N}(x,y):=\left[7+\mathsf{e}^{x}\right] \quad\text{and}\quad \mathcal{M}(x,y):=\mathsf{e}^{x}\left[y-2x\right].$$

Its soln $y=y(x)$ satisfies $\mathbf{F}\big(x,y(x)\big)=\text{Const}$, where $\mathbf{F}(x,y)=$.

b DE $\left[2xy\cdot\frac{\mathrm{d}y}{\mathrm{d}x}\right]+[2+3x]y^2=0$ is <u>not</u>, alas, *exact*. Happily, multiplying both sides by (non-constant) fnc

$W(x)=$

gives a *new* DE which *is* exact. **Did you *Check?***

c *A* soln to $\left[f''+f'\right](x)=2x$ is **polynomial** $f(x)=$. Using parameters α and β, then, the *general* solution to $\left[h''+h'\right](x)=2x$ is $h_{\alpha,\beta}(x)=$.

And the h with $h(0)=1$ and $h'(0)=0$ is $h(x)=$.

d A set $\mathcal{S}:=\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$ of vectors is ***linearly independent*** if [Imagine 4 lines on which to write].

e A fnc $g(x,y,z)$ is ***homogeneous*** if

[Imagine 4 lines on which to write].

f Function $g(t,x):=\cos(t^2+x^2)/[t^2+x^2]$ is *homogeneous*: [Circle True or False] *T* *F*

g Matrix-product $\begin{bmatrix}2&-3\\4&5\end{bmatrix}\cdot\begin{bmatrix}7&w\\2&1\end{bmatrix}=$.

With $\mathsf{A}:=\begin{bmatrix}2&-3\\4&5\end{bmatrix}$, then the $(2,1)$-entry of A^{-1} is .

h The square-roots of $2\boldsymbol{i}$ are $\pm$.

A particular cube-root of $64\boldsymbol{i}$ is $R\cdot\mathsf{e}^{i\theta}$, for positive reals $R=$ and $\theta=$.

i In $\mathbb{R}$: $[1+\boldsymbol{i}]^{86}=\left[\quad\right]+\boldsymbol{i}\cdot\left[\quad\right]$.

[*Hint: Multiplying complexes multiplies their modulii, and adds their angles. You may use* sin *and* cos *if you wish.*]

OYOP: *In grammatical English* ***sentences****, write your essay on every* ***third*** *line* (usually)*, so that I can easily write between the lines. Do* **not** *restate the question.*

P2: Brine with $5\frac{\mathsf{lb}}{\mathsf{gal}}$ salt flows at rate $1\frac{\mathsf{gal}}{\mathsf{min}}$ *into* a tank that initially held 100gal of $2\frac{\mathsf{lb}}{\mathsf{gal}}$-salt brine. The tank is well-mixed, and brine is flowing *out* at rate $3\frac{\mathsf{gal}}{\mathsf{min}}$. So the tank will empty in minutes.

At time t, let $\sigma(t)$ denote the tank-salinity [in lb/gal] and use $y(t)$ for the total number of pounds of salt in the tank.

Explain how to derive a DE for $y()$. Don't just pull a DE out of the air; **explain**, using *Text* and *Pictures*, <u>how</u> it comes from the physical situation. Now re-write the DE in linear-DE form. Use FOLDE to solve the DE. Dividing by the amount of water in the tank at time t, gives this formula for the salinity:

$\sigma(t)=\left[\quad\right]\frac{\mathsf{lb}}{\mathsf{gal}}$.

As the tank approaches empty, its salinity approaches [ ] $\frac{\text{lb}}{\text{gal}}$. *Do Not approximate.*

P3: Fnc $y = y(t)$ solves DE

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{3t - y + 1}{t + y + 3}\,.$$

Showing all the steps, write an essay deriving an explicit or implicit soln $y_\alpha(t)$.