



Staple!

Linear Algebra
MAS4105 5441**B-Home**Prof. JLF King
Touch: 6May2016

Hello. Your essays violate the CHECKLIST at *Your Peril!* Page numbers without citation refer to our textbook. Each member of the team must retain a *complete copy* of the team's problem-sheet and write-up, including diagrams. Exam is due **by 4:30PM Tues, 01Nov2005**. Write expressions unambiguously e.g., "1/a + b" should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with **negative** signs!)

B1: Short answer: Show no work. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

z Professor King sometimes gives freebie questions. Circle one: **True** **Right On!** **Who?**

a Let $R := \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ 0 & 1 \end{bmatrix}$. Find a particular left-inverse L to R ; this left-inverse, L , should have its leftmost column all-zero. (Of nec., L is a 2×3 matrix.)

$L = \dots$ and $LR = \dots$

b Construct 2×2 matrices $A \neq 0$ and non-sing Q so that $QAQ^{-1} = 0$. (Of nec., the determinant of A is 0.)

$A = \dots$ and $Q = \dots$

c+ Let $U: \text{Poly}_{<3} \rightarrow \text{Poly}_{<3}$ by $U(f) := 2f'' - f' + f$. Polynomials

$$b_0(x) := 1, \quad b_1(x) := x + 3, \quad b_2(x) := x^2 + 1,$$

yield a basis $\mathcal{B} := (b_0, b_1, b_2)$. Compute the matrix

$$[U]_{\mathcal{B}}^{\mathcal{B}} = \dots \text{ Rank}(U) = \dots$$

d **#4(f) P237.** (Jog: Det of complex matrix.)
Det = \dots

e+ The char.poly of square matrix S is

$$p_S(x) := [x - 2]^{52} + [x + 3]^{45}.$$

So $\text{Trace}(S) = \dots$ and $\text{Det}(S) = \dots$

B2: Consider 2×2 matrices A and B , satisfying $[A + B]^2 = 0$. Suppose $A^2 = \begin{bmatrix} 31 & 12 \\ 18 & 7 \end{bmatrix}$ and $B^2 = \begin{bmatrix} 29 & 12 \\ 12 & 5 \end{bmatrix}$ and $AB = \begin{bmatrix} -29 & -12 \\ -17 & -7 \end{bmatrix}$. Compute, easily, the product matrix $A^{-1}B^{-1} = \dots$ [Hint: Use matrix algebra.]

B3: Carefully prove **#22P109.** Then use it to prove **#22P230.**

Team: _____

B4: Let $G := \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$. In std.form, its char.poly is $p_G(x) = \dots$. Counting with multiplicity, $\text{Evals}(G) = \dots$. Either show that G is not diagonalizable, or else compute diagonal matrix

$D = \dots$ and non-sing $Q = \dots$ so that $QDQ^{-1} = G$.

B5: Please give a rigorous proof of **#24P230.** [Hint: Try induction on n , using either the cofactor, or general-diagonal, or rref method of computing determinants.]

End of B-Home

B1: 120pts

B2: 85pts

B3: 185pts

B4: 105pts

B5: 45pts

Total: 540pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)." Name/Signature/Ord*

Ord:

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