Staple!

Team B_____

Number Theory MAS4203 | **Home-B** | Prof. JLF King *Touch*: 2Jul2018

Hello. Essays violate the CHECKLIST at *Grade Peril*! Exam is due by **9:30*AM*, Monday, 27Mar2006**.

B1: Short answer: Show no work. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

z In Fall2006, there will be a continuation NT course covering topics in Cryptography, Primality testing, Factorization algorithms, Diophantine eqns and other topics of chosen by students. Circle one: **True Oui Da Yes** *Uh, Did I miss something??*

a $y=$ _____, where $8y \equiv_{15} 7$, $17y \equiv_{21} 13$, $4y \equiv_{35} 1$. [For next few probs, consider "$\text{Gcd}(M_j, M_i) \nmid [T_j - T_i]$" method.]

b There are $N=$ _____ eggs in a basket. If the eggs are removed from the basket 2, 3, 4, 5 and 6 at a time, there remain 1, 2, 3, 4 and 5 eggs in the basket, respectively. Were the eggs removed 7 at a time, none would remain. (Brahmagupta, circa 600 A.D.)

c Use CRT to produce all RONOs (*square* Root Of Negative One), modulo 1105. [*Hint:* $1105 = 5 \cdot 13 \cdot 17$.]
RONOs= _____ $\in [1 \,..\, 1105)$.

B2:

i Show all steps, except the $\not\downarrow$ tables, to compute a magic tuple $\mathbf{G}$ so that $g: \mathbb{Z}_5 \times \mathbb{Z}_6 \times \mathbb{Z}_7 \to \mathbb{Z}_{210}$ is a ring-isomorphism, where

$$g\big((z_1, z_2, z_3)\big) \;:=\; \Big\langle z_1 G_1 + z_2 G_2 + z_3 G_3 \Big\rangle_{210} .$$

ii Consider poly $h(x) := [x-2][x-32][x-8]$. Find all solutions to congruences $h(x) \equiv_M 0$, for $M = 5, 6, 7$, displaying the *results* in a nice table. (Do **not** show work for this step.)

Now use your ring-iso to compute *all* solns x to $h(x) \equiv_{210} 0$, displaying the results in a table which shows *which* 3tup each came from. There are (not counting multiplicities) K:= _____ many solns.
Explain your method well; then show **one** computation giving a root *different* (mod 210) from 2, 32, 8.

B3: Solve LeVeque's #7[P]63, but rename a, b, n to A, B, N.

B4: α Give an explicit soln with $\vec{R} := (4, 1, 2, 3)$.

β Five guys on a deserted island have already collected a lotta coconuts. On consecutive days $k = 0, 1, 2, \ldots, N$, a guy gets up early, throws R_k many (a natnum) coconuts to the monkey, and the remaining pile miraculously divides evenly by 5. He hides one-fifth, and pushes the remaining 4 piles together.

With knowledge of the ***remainder sequence***

$$\vec{R} \;=\; (\mathsf{R}_1, \mathsf{R}_2, \ldots, \mathsf{R}_N) \;:$$

How many coconuts in the original pile P_0?

End of Home-B

B1:	___ ___	80pts
B2:	___ ___	95pts
B3:	___ ___	95pts
B4:	___ ___ ___	120pts
Total:	___ ___ ___	390pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: _____

Ord: _____

Ord: _____