Staple!

Team B___

Sets and Logic
MHF3202 1079

Home-B

Prof. JLF King
Wedn., 18Oct2023

Due **BoC, Monday, 23Oct2023**, wATMP! **Print this problem-sheet**; it is the first page of your write-up, with the blanks filled in (handwritten). Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$. [Put ordinal, Team-# and sign Honor Code.]

B1: *Show no work.*

__ __ **a** For a finite list $\mathcal{S}$ of posints, define

$$\boldsymbol{\mu}_{\mathcal{S}}(N) := \Big\{ k \in [1..N] \;\Big|\; \exists d \in \mathcal{S} \quad \text{with} \quad d \bullet\!| \; k \Big\}.$$

For $\mathcal{S} := \{6, 7, 10\}$, the Inclusion-Exclusion formula (using the floor fnc) for the number of elements, $|\boldsymbol{\mu}_{\mathcal{S}}(N)|$, is

. . . .

(Write your answer, using the floor function as appropriate, in form [term + term + term] − [term + term + term] + term.
The terms are computed from the $\{6, 7, 10\}$ numbers.)

When $N := 67$, then, $|\mu_{\{6,7,10\}}(67)| =$ . . . .

__ __ **b** Let $\mathcal{P}_\infty$ denote the family of all ***co-finite*** subsets of $\mathbb{N}$. That is, a subset $S \subset \mathbb{N}$ is an *element* of $\mathcal{P}_\infty$ IFF $\mathbb{N} \setminus S$ is finite. Define relation $\bowtie$ on $\mathcal{P}_\infty$ by: $A \bowtie B$ IFF $A \cap B$ is infinite.

Stmt "*This $\bowtie$ is an equivalence-relation*" is: *T* *F*

__ __ **c** Suppose that $\prec$ is a total-order on set $\mathcal{S}$, and $<$ is total-order on set $\boldsymbol{\Omega}$, both strict. Define binrel $\lll$ on $\mathcal{S} \times \boldsymbol{\Omega}$ by:

$$(b, \beta) \lll (c, \gamma)$$

IFF *Either* $b \prec c$ *or* $[b = c$ and $\beta < \gamma]$.

Then:
Relation $\lll$ is a total-order. *T* *F*

Suppose $\prec$ and $<$ are each well-orders.

Then $\lll$ is a well-order. *T* *F*

Carefully TYPE *your two essays, double–spaced. I suggest* LaTeX, *but other systems are ok too.*

B2: Let $\mathbf{E}_n$ be the equilateral triangle with side-length 2^n. This $\mathbf{E}_n$ can be tiled in an obvious way by 4^n many little-triangles [copies of $\mathbf{E}_0$]; [picture on blackboard] The ***"punctured*** $\mathbf{E}_n$**"**, written $\widetilde{\mathbf{E}_n}$, has its topmost copy of $\mathbf{E}_0$ removed.

A ***(trape)zoid***, $\mathbf{T}$, comprises three copies of $\mathbf{E}_0$ glued together in a row, rightside-up, upside-down, rightside-up [picture on blackboard] [A ***zoid-tiling*** allows all six rotations of $\mathbf{T}$.]

i Prove: *For each n, board $\widetilde{\mathbf{E}_n}$ admits a zoid-tiling.*

ii Let Δ_k be the equilateral triangle of sidelength k; so $\mathbf{E}_n$ is Δ_{2^n}. Triangle Δ_k comprises k^2 little-triangles.

For what values of k does Δ_k admit a zoid-tiling?

For which k does $\widetilde{\Delta_k}$ admit a zoid-tiling?

iii *Do something non-trivial, extra.* E.g: Do the results extend with the punture anywhere? Can you count the # of tilings of a given region? Are there regions admitting non-isomorphic tilings?

B3: Prove, for each natnum N, that

$$\sum_{k=0}^{N} \left[\binom{N}{k}^2 \right] = \binom{2N}{N}.$$

[Can use Double-counting, or Induction.]

b *Generalize this problem non-trivially.*

B1:	__ __ __	115pts
B2:	__ __ __	110pts
B3:	__ __	70pts
Total:	__ __ __	295pts

Honor Code: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* Name/Signature/Ord

Ord:
. . . .

Ord:
. . . .

Ord:
. . . .