Staple!

Team: B

Sets and Logic
MHF3202 3E07

Home-B (v.g3)

Prof. JLF King
Wedn., 11Mar2020

Due [was: Monday, 16Mar2020, 10PM] (extended to) **Tuesday, 17Mar2020**, by **10AM**. Each team emails me (squash@ufl.edu) one PDF. The format *must* be a PDF.

Please *fill-in* every *blank* on this sheet. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

B1: *Show no work.*

___ ___ a) Compute the real α= .... such that

$$*: \quad 3^\alpha \cdot \sum_{k=0}^{4000} \binom{4000}{k} 2^k = \sum_{j=0}^{1995} \binom{1995}{j} 8^j .$$

[*Hint:* The Binomial Theorem]

___ ___ b) On $\Omega := [1..29]\times[1..29]$, define binary-relation **C** by: $(x,\alpha)\,\mathbf{C}\,(y,\beta)$ IFF $x\cdot\beta \equiv_{30} y\cdot\alpha$. Statement "*Relation* **C** *is an* ***equivalence relation***" is: *T* *F*

___ ___ c) Circle those operators/relations which are chiral:

$\neq$ $\bullet\!|$ $\circ$ Max $\div$ $\leq$ $<$ $\wedge$

___ ___ d) On a K-elt set Ω, the number $\#_K$ of **reflexive symmetric** binrels is ....

In particular, $\#_5 =$ ....

On a 3-set, there are .... many equiv.relations.

For the three essay questions, carefully TYPE, *double spaced, grammatical solns. I suggest* LaTeX, *but other systems are ok too.*

B2: On a 7×7 chessboard, 23 rooks are placed. Prove: ***Either*** there exists a friendly 5-set, *or* a *disjoint-pair* of friendly 4-sets. [An n-set of rooks is ***friendly*** if the rooks lie on n distinct rows, and n distinct columns. Shorthand: You may use ***double-clump*** for "disjoint-pair of friendly 4-sets".] [*Hint:* PHP]

Stronger: *Prove there always exists a double-clump.*

B3: Prove, for each natnum N, that

$$\sum_{k=0}^{N} \left[\binom{N}{k}^2\right] = \binom{2N}{N}.$$

[Can use Double-counting, or Induction.]

B4: Give a careful bijective proof of:

Thm: *Fix a natnum $N \geq 3$. Then*

$$*: \quad [\![N\downarrow 3]\!]\cdot 2^{N-3} = \sum_{k=3}^{N} [\![k\downarrow 3]\!]\cdot\binom{N}{k}.$$

[Can you also prove this by induction on N?]

End of Home-B

B1:	___ ___ ___	115pts
B2:	___ ___ ___	125pts
B3:	___ ___	50pts
B4:	___ ___	50pts
Total:	___ ___ ___	340pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* Name/Signature/Ord

.... Ord: ....

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