Due   **BoC, Monday, 22Oct2018**,   wATMP!
Please *fill-in* every *blank* on this sheet.   Write
DNE in a blank <u>if</u> the described object does not exist
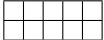or if the indicated operation cannot be performed.

B1: *Show no work. Simply fill-in each blank on the problem-sheet.*

——— a On a K-element set, the
number of reflexive symmetric binrels is ⌊. . . . .⌋ .

On a 4-set, there are ⌊. . . . .⌋ many equiv.relations.

——— b On $\mathbb{Z}_+$, write $x \$ y$ IFF $\text{GCD}(x,y) \geqslant 2$. So $\$$ is
Circle

Transitive: $T\ F$.   **Symm.**: $T\ F$.   **Reflex.**: $T\ F$.

On $\mathbb{Z}$, say that $x \nabla y$ IFF $x - y < 1$. Then ∇ is:

Trans.: $T\ F$.   **Symm.**: $T\ F$.   **Reflex.**: $T\ F$.

——— c On $\Omega := [1 .. 29] \times [1 .. 29]$, define binary-relation $\mathbf{C}$ by:
$(x, \alpha)\, \mathbf{C}\, (y, \beta)$ IFF $x \cdot \beta \equiv_{30} y \cdot \alpha$.   Statement "*Relation* $\mathbf{C}$
is an **equivalence relation**" is: $T\ F$

For the two essay questions, carefully TYPE, *double-
or-triple–spaced, grammatical solns.*

B2: Consider board $\mathbf{B}_N := 2 \times N$; so [grid] is $\mathbf{B}_5$.
Use T_N for the number of tilings of $\mathbf{B}_N$ by 1×1 ("1-minos")
and 2×1, 1×2 ("dominos"). Evidently $T_0 = 1$ and $T_1 = 2$.

I   PROVE: *Each natnum N satisfies*

∗: $$T_{N+2} = T_N + 2\sum_{j=0}^{N+1} T_j\,.$$

In addition to your essay, show your ideas in pictures.

II   Derive a Fibonacci-like CCLRR

$T_{N+3} =$ ⌊. . . . .⌋ .

So $T_7 =$ ⌊. . . . .⌋ , and $T_n = \alpha A^n + \beta B^n + \gamma C^n$, for some
numbers α, β, γ, where A, B, C are roots of polynomial

$f(x) =$ ⌊. . . . .⌋ .

For large n, then, $T_n \approx \alpha A^n$, where [decimal approxima-
tion] $A \approx$ ⌊. . . . .⌋ and $\alpha \approx$ ⌊. . . . .⌋ .

B3: In our Velleman text, solve problem **#12**$^{P.}$*277*. Let $\mathbf{E}_n$ be the equilateral triangle with side-length 2^n. This $\mathbf{E}_n$ can be tiled in an obvious way by 4^n many little-triangles [copies of $\mathbf{E}_0$]; see picture P.277. The "***punctured* $\mathbf{E}_n$**", written $\widetilde{\mathbf{E}}_n$, has its topmost copy of $\mathbf{E}_0$ removed.

A (***trape***)***zoid***, **T**, comprises three copies of $\mathbf{E}_0$ glued together in a row, rightside-up, upside-down, rightside-up [picture P.277]. [A ***zoid-tiling*** allows all three rotations of **T**.]

> **i**
>
>   PROVE: *For each n, board $\widetilde{\mathbf{E}}_n$ admits a zoid-tiling.*

> **ii**
>
>   Let Δ_k be the equilateral triangle of sidelength k; so $\mathbf{E}_n$ is Δ_{2^n}. Triangle Δ_k comprises k^2 little-triangles.
> *For what values of k does Δ_k admit a zoid-tiling?*
> *For which k does $\widetilde{\Delta}_k$ admit a zoid-tiling?*

> **iii**
>
>   An ***Lmino*** (pron. "ell-mino") comprises three squares in an "L" shape (all four orientations are allowed).
> Let $\mathbf{S}_n$ be the $2^n{\times}2^n$ square board, comprising 4^n ***squaries*** (little squares). Have $\widetilde{\mathbf{S}}_n$ be the board with one corner squarie removed. Velleman inductively shows, pp.272-275, that each $\widetilde{\mathbf{S}}_n$ is Lmino-tilable (by $[4^n-1]/3$ Lminos, of course). Further, with $\mathbf{S}'_n$ denoting $\mathbf{S}_n$ with an *arbitrary* puncture, V. proves that every $\mathbf{S}'_n$ is Lmino-tilable.

     Generalize this to three-dimensions. Let $\mathbf{C}_n$ denote the $2^n{\times}2^n{\times}2^n$ cube, $\widetilde{\mathbf{C}}_n$ the corner-punctured cube, and let $\mathbf{C}'_n$ be $\mathbf{C}_n$ but with an arbitrary ***cubie*** removed.
     What is the 3-dimensional analog of an Lmino? Calling it a **"3-*mino*"**, how many cubies does it have? [Provide a drawing of your 3-mino.] PROVE: *Every $\mathbf{C}'_n$ admits a 3-mino–tiling.* [Provide also pictures showing your ideas.]

> **iv**
>
>   Generalize to K-dim(ensional) space, with $\mathbf{C}_{n,K}$ being the $2^n{\times}.\!\!\overset{K}{.}\!\!.{\times}2^n$ cube, having $[2^n]^K=2^{nK}$ many K-dim'al cubies. As before, let $\mathbf{C}'_{n,K}$ be $\mathbf{C}_{n,K}$ with an arbitrary cubie removed.
>   *What is your K-**mino** with which you will tile, and how many cubies does it have?* (So a 2-mino is our Lmino.) PROVE: *Every $\mathbf{C}'_{n,K}$ admits a K-mino–tiling.*

B1:	___ ___	95pts
B2:	___ ___	80pts
B3:	___ ___ ___	155pts
Total:	___ ___ ___	330pts