Staple!

Team: B

Abstract Algebra
MAS4301 3175

Home-B

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Hello. Essays violate the CHECKLIST at *Grade Peril*! Exam is due by **noon, Thursday, 14Feb2008**, slid under my office door, LIT402. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq$ {} $\neq 0 \neq$ *Empty-word*..

Let F and R be the ***flip*** and ***rotation*** in the dihedral group $\mathbb{D}_N$, with F^2=**e**, R^N=**e** and RFRF=**e**. Use R^j and R^jF as the standard form of each element in $\mathbb{D}_N$.

Use $\mathbb{Y}_N$ or $\mathbb{Y}(N)$ to denote the cyclic group of order N.

Fill-in *all* blanks (*handwriting; don't bother to type*) on this sheet **including** the blanks for the essay questions!

B1: Show no work.

___ ___ **a** With $N := 36300 = 5^4 \cdot 2^8 \cdot 11^3$, the longest *chain* of subgroups

$$\{\varepsilon\} \subsetneq \langle g_1 \rangle \subsetneq \langle g_2 \rangle \subsetneq \ldots \subsetneq \langle g_K \rangle = \mathbb{Y}_N$$

has $K =$ .... [*Hint:* No OBO!]

And $\mathbb{Y}_N$ has .... subgroups.

___ ___ **b** Euler $\varphi(36300) =$ ....

Express your answer as a product ${p_1}^{e_1} \cdot {p_2}^{e_2} \cdot \ldots$ of *primes* to posint powers, with $p_1 < p_2 < \ldots$.

___ ___ **c** Mod K:=4301, the recipr. $\langle \frac{1}{237} \rangle_K$= .... $\in [0..K)$.

[*Hint:* ⸸] So x= .... $\in [0..K)$ solves $4 - 237x \equiv_K 1$.

___ ___ **d** In $\mathbb{S}_4$, the subgp, H, generated by $y := (1\ 2)(3\ 4)$ and $z := (2\ 4\ 3)$ has .... many elements.

___ ___ **e** $G := (\mathbf{U}(23), \cdot, 1)$ is cyclic. The smallest generator is ...... $\in [2..21]$. And G has .... many generators.

___ ___ **f** In $\mathbb{S}_{11}$, the maximum possible order of an element is MaxOrd($\mathbb{S}_{11}$) = LCM(...............) = ...

___ ___ **g** Here are two elements in $\mathbb{S}_{11}$:

$$\alpha := (1\ 2\ 3)(4\ 5\ 6\ 7)(8\ 9\ 10\ 11) \ ;$$
$$\beta := (1\ 3\ 2)(4\ 5\ 7\ 6)(8\ 9\ 11\ 10) \ .$$

Both α and β are in $\mathbb{A}_{11}$:	**True**	**False**
Elts α and β are conjugate in $\mathbb{S}_{11}$:	**True**	**False**
Elts α and β are conjugate in $\mathbb{A}_{11}$:	**True**	**False**

___ ___ **h** As a product binomial-coeffs and factorials, the number of order-15 elts in $\mathbb{S}_8$ is ....

i After first shuffle, **#50**^{P}*112*, the cards were in order
....

j+ In $\mathbb{S}_4$, the centralizer of $q := (1\ 2)(3\ 4)$ has ...... many elts. In $\mathsf{C}(q)$, the number of elements of each cycle-signature is: $\lceil 1^4 \rfloor$: .... $\lceil 1^2, 2^1 \rfloor$: .... $\lceil 1^1, 3^1 \rfloor$: .... $\lceil 2^2 \rfloor$: .... $\lceil 4^1 \rfloor$: ....

Essay questions: **Fill-in all blanks.** *For each question, carefully type a double– or triple–spaced essay solving the problem. Each essay starts a new page.*

B2: Produce (with proof, natch') a finite group G and explicit elts $\boldsymbol{x},\boldsymbol{y} \in G$ with *different prime* orders $p \neq q$, so that $\mathrm{Ord}(\boldsymbol{xy}) \perp p \cdot q$. [*Hint:* Necessarily, $\boldsymbol{x} \not\approx \boldsymbol{y}$.]

B3: Group $\mathbb{D}_5$ has .... many automorphisms of which .... are inner-auts. Exhibit an *outer*-aut, defined by $\alpha(R)$:= .... and $\alpha(F)$:= .... [Use form $R^J F^K$.] Prove that your defn extends to an automorphism. Prove that your α is **not** an inner-automorphism.

End of Home-B

B1:	___ ___ ___	155pts
B2:	___ ___	35pts
B3:	___ ___	75pts
Total:	___ ___ ___	265pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ....

Ord: ....

Ord: ....