



Staple!

Algebra 1
MAS4301 3175 **Home-B** Prof. JLF King
Monday 31Mar2003

Hello. The order of your hand-in should be: PROBLEM SHEET (P-S, this side up), TYPESETTING CONVENTIONS (if any), followed by the write-up to each question. General instructions are on the CHECKLIST. *Each member* of the team must retain a *complete copy* of the team's essays, **including diagrams**.

Use " $f(x)$ notation" when writing fncs; in partic., for trig and log fncs. E.g, write " $\sin(x)$ " rather than $\sin x$ or $[\sin x]$. Be sure to write unambiguously, e.g, " $1/a + b$ " should be bracketed either $[1/a] + b$ or $1/[a + b]$.

B1: Please solve #22Page225. (*Jog:* The pre-image of 17.)

B2: Prove #24Page236. (*Jog:* The units group of a direct-sum of commutative rings.)

B3: Solve #49Page249. (*Jog:* Find all solns to a quadratic eqn over a finite ring.)

B4: Prove #5Page260. (*Jog:* Is this subring an ideal?)

B5: Recall, for N a posint, the dihedral group $\mathbb{D}_N = \langle F, R \rangle$ with presentation

$$F^2 = R^N = [F \cdot R]^2 = \epsilon.$$

Draw a (large!) Cayley digraph $\text{Cay}(\mathbb{D}_N; \{F, R\})$.

Compute the center $Z(\mathbb{D}_N)$. Give a complete description –and compute the cardinality of– groups $\text{Inn}(\mathbb{D}_N)$ and $\text{Aut}(\mathbb{D}_N)$. What are the orders of elements in these groups, and how many elements of each order?

i For what values of N can you identify (up to isomorphism) group $\text{Inn}(\mathbb{D}_N)$? $\text{Aut}(\mathbb{D}_N)$?

ii For what values of N can you identify $\text{Out}(\mathbb{D}_N)$, the quotient group of outer automorphisms?

B6: Solve the “*Coconuts*” problem. First solve the case where there is a coconut left-over the next morning. If the divide by 5 with one left over operation is done N times, leaving a posint amount, then what was the starting number?

a Find an interesting generalization of the problem,

b Find a version of the problem appropriate for the in-class part.

Team: _____

B1:	_____	40pts
B2:	_____	50pts
B3:	_____	50pts
B4:	_____	45pts
B5:	_____	135pts
B6:	_____	135pts

Total: _____ 455pts

HONOR CODE: *“I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague).”* **Name/Signature/Ord**

Ord: _____

Ord: _____

Ord: _____