Staple!

Sets and Logic
MHF3202 12A3

Project-B

Prof. JLF King
Sunday, 21Nov2021

Welcome! The IPB (Individual Project B) is due BoC (Beginning of Class) on Monday, 29Nov2021. It must be typeset.

B1: Each dot of W many, gets one of 4 colors. The minimum W *guaranteeing* that at least 3 dots have the same color is $W =$ .... *Prove* your answer, and show that $W-1$ is insufficient.

With this W, the ***W×H-grid*** is $\mathbf{G} := [1..W]\times[1..H]$, for an H you will determine. A subset $\mathbf{S} \subset \mathbf{G}$ of form

$$\mathbf{S} := \{x_1, x_2, x_3\} \times \{y_1, y_2\}$$

where $x_1 < x_2 < x_3 \leq W$ and $y_1 < y_2 \leq H$ are positive integers, is a ***3×2-subgrid*** of $\mathbf{G}$. The minimum H *guaranteeing* that each 4-coloring of $\mathbf{G}$ admits a *monochromatic* 3×2-***subgrid*** is $H =$ .... *Prove* that your H is sufficient. Prove that $H-1$ is *not* sufficient.

a Now allowing C many colors, compute the corresponding values W_C and H_C. (This C was called "N" on the take-home.)

b Can you generalize to 3 dimensions? Further?

B2: i Over a 29 day month, SeLoidian Bubba posts at least one soln per day, for a total of 45 solns. PROVE:

> *There is a period of consecutive days over which he posted exactly $g := 13$ solutions.*

[g for "**G**uaranteed".] NOTE: In your proof, let $\mathbf{s}_n$ denote the number of solns posted that month by the end of day n. By hyp., then,

$$1 \leq \mathbf{s}_1 < \mathbf{s}_2 < \ldots < \mathbf{s}_{29} = 45\,.$$

Let $\mathbf{t}_n := 13 + \mathbf{s}_n$. *Using this notation, write a complete, rigorous proof, proving any lemmas you need/want.* [*Hint:* You may find it easier to first show that $g{=}12$ is guaranteed. Then you'll see how to show that $g{=}13$ is guaranteed.]

ii *Generalize*: Replace 29 by D, replace 45 by P; we now consider posints with D < P . Give a formula for the *largest* value, call it $\Gamma(\mathsf{D},\mathsf{P})$, for which your proof *guarantees* the values $g = 1, 2, 3, \ldots, \Gamma(\mathsf{D},\mathsf{P})$.

(*Stuff commed-out.*)

iii For fixed D and P, let $\mathcal{M}_{\mathsf{D}}^{\mathsf{P}}$ be the *set* of guaranteed posints g. What can you tell me about the structure of $\mathcal{M}_{\mathsf{D}}^{\mathsf{P}}$? Conjectures? Proofs? Computer experiments? (I don't know the structure. What can you teach me?)

B3: Sequence $\vec{\mathbf{q}} = (q_0, q_1, \ldots)$ is defined recursively by

$$*: \qquad q_{n+1} := \frac{[q_n]^2 + 5}{q_{n-1}}\,,$$

with $q_0 := 2$ and $q_1 := 3$.

Prove that $\vec{\mathbf{q}}$ can be continued backwards. Prove each q_n is an integer. Compute the $\lim_{n\to\infty} q_{n+1}/q_n$.

What else can you tell me about $\vec{\mathbf{q}}$?

B1:	___ ___	85pts
B2:	___ ___ ___	145pts
B3:	___ ___	85pts
Total:	___ ___ ___	315pts

NAME: ....

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....