Linear Alg
MAS3114 2481

Class-B

Prof. JLF King
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For questions B1 and B2 please show no work. Do **not** hand in "scratch work"; only hand in carefully written work for problem B3. Use $\mathbf{0}$ and $\mathbf{I}$ to mean $\mathbf{0}_{2\times 2}$ and $\mathbf{I}_{2\times 2}$, respectively.

Do not make approximations. Write expressions unambiguously e.g, "$1/a + b$" should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with **negative** signs!)

B1: Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed. (...or if the described matrix does not exist.)

z Prof. King maintains an email archive for his students. Circle one: **True** **Yes** ***He does?***

a The inverse of product $\mathsf{BCB}^3\mathsf{E}^{-1}\mathsf{C}$ of invertible 7×7 matrices is ___ .

AAa Over $\mathbb{Q}$, the inverse of $\mathsf{E} := \begin{bmatrix} 1 & x & z \\ & 1 & y \\ & & 1 \end{bmatrix}$ is

Note. For the following, $\mathsf{A}, \mathsf{B}, \mathsf{C}, \mathsf{D}, \mathsf{E}$ represent 2×2 matrices. Give an example of...

c ...matrices C, D which are invertible, but $\mathsf{C}+\mathsf{D}$ is not. $\mathsf{C} =$ ___ and $\mathsf{D} =$ ___ .

d ...matrices $\mathsf{A} \neq \mathsf{B}$ for which $\mathsf{A}^3 = \mathsf{B}^3$.
$\mathsf{A} =$ .... and $\mathsf{B} =$ ....

e ...matrix C which is zero-going (i.e, $\mathsf{C}^n \to \mathbf{0}$ as $n \to \infty$) but not nilpotent. $\mathsf{C} =$ ___ .

f ...matrix D which is nilpotent but not singular. $\mathsf{D} =$ ___ .

g ...matrix E which is singular (i.e, non-invertible) but not zero-going. $\mathsf{E} =$ ___ .

B2: **h** Consider 2×2 matrices $\mathsf{B}, \mathsf{A}, \mathsf{D}$ with A&D invertible. Let M be the partitioned matrix $\left[\begin{smallmatrix} \mathsf{A} & \mathsf{B} \\ \mathbf{0} & \mathsf{D} \end{smallmatrix}\right]$. (So M is 4×4.) Then M is invertible and M^{-1} has the partitioned form $\begin{bmatrix} ? & \mathsf{E} \\ \mathbf{0} & ? \end{bmatrix}$, where the "?" are various 2×2 matrices
and $\mathsf{E} =$ ___ .
(Express E in terms of $\mathsf{A}, \mathsf{B}, \mathsf{D}$, their inverses, and $\mathbf{I}$.)

i Let $\mathsf{D} := \left[\begin{smallmatrix} 1 & \\ & 2 \end{smallmatrix}\right]$. Then $\mathsf{D}^{2481} =$ ....
Let $\mathsf{A} := \begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$. Observe that $\mathsf{A} = \mathsf{P}\mathsf{D}\mathsf{P}^{-1}$, where $\mathsf{P} := \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$. Then $\mathsf{A}^{2481} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
where $b =$ ....
(Express b in terms of 2^{2481} and other specific numbers.)

B3: *On separate sheets of paper, show all work for this problem, writing in complete sentences that end with visible periods.* (Staple your write-up to the back of this problem sheet.)

Suppose θ is an angle which is neither 0 Deg nor 180 Deg, and let $c := \cos(\theta)$ and $s := \sin(\theta)$. Let R denote the "rotation by angle θ" matrix. In terms of letters "c" and "s", find formulas for the three numbers
$z =$ ___ , $y =$ ___ , $x =$ ___ ,
so that $\mathsf{U}_z\mathsf{L}_y\mathsf{U}_x = \mathsf{R}$. Recall that U_α denotes the Upper triangular matrix $\left[\begin{smallmatrix} 1 & \alpha \\ 0 & 1 \end{smallmatrix}\right]$; a horizontal shear. Similarly, L_β is $\left[\begin{smallmatrix} 1 & 0 \\ \beta & 1 \end{smallmatrix}\right]$; a vertical shear.

End of Class-B

B1:	___ ___ ___	240pts
B2:	___ ___	80pts
B3:	___ ___	65pts
Total:	___ ___ ___	385pts

Please PRINT your name and ordinal. Ta:

.... Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....