

Sets and Logic
MHF3202 1079

Class-B

Prof. JLF King
Wedn, 25Oct2023

B4: Short answer. Show no work. Write LARGE.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: $\text{DNE} \neq \{\} \neq 0$.

a For a LOR (letter-of-recommendation), Prof. K requires two courses, or a Special Topics or graduate course Circle:

Yes True Darn tootin'!

b Mimicking what we did in class: From the 987×200 game-board, cut-out (remove) the $(35, 150)$ -cell and one other cell at $P = (x, y)$. Circle those choices for P ,

$(150, 160)$, $(14, 35)$, $(66, 77)$, $(195, 15)$, $(123, 4)$

which, if removed, would leave a board that *definitely cannot* be domino-tiled.

c Let \mathcal{P}_∞ denote the family of all *infinite* subsets of \mathbb{N} . Define relation \approx on \mathcal{P}_∞ by: $A \approx B$ IFF $A \cap B$ is infinite. Stmt "*This \approx is an equivalence-relation*" is: T F

d On a 3-set, there are _____ many equiv.relations. [...]

e Let δ_N be the number of derangements of $[1..N]$. Written in Incl-Excl notation (the formula we derived in class), $\delta_{15} =$ _____

Using binom-coeffs and derangements, the number of N -perms with precisely

3 fixed-points is: _____

[You may use binom-coeffs and $\delta_1, \delta_2, \dots$ in your answer.]

f A $k \in [1..100]$ is *good* if $k \mid 2$ or $k \mid 3$ or $k \mid 5$. So #Good = _____ [Hint: Inclusion-exclusion]

OYOP: In grammatical English *sentences*, write your essay on every 2nd line (usually), so I can easily write between the lines.

B5: An *Lmino* (pron. "ell-mino") comprises three squares in an "L" shape (all four orientations are allowed). For natnum N , let \mathbf{R}_N denote the $3 \times N$

board: I.e.,

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| | | | |
| | | | |

 is the \mathbf{R}_5 board. Prove:

Theorem: When N is odd, then board \mathbf{R}_N is not Lmino-tilable.

You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, *labeled* pictures.

When N is *even*, our \mathbf{R}_N has exactly _____ many Lmino-tilings.

B4: _____ 120pts

B5: _____ 55pts

Total: _____ 175pts

NAME: _____ Ord: _____

HONOR CODE: "*I have neither requested nor received help on this exam other than from my professor.*"

Signature: _____