Sets and Logic MHF3202 **Class-B** Prof. JLF King Wedn., 17Feb2021

B1: Short answer. Show no work.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

10 10 a Using *only* symbols $\boldsymbol{P}, \boldsymbol{Q}, \wedge, \vee, \neg, T, F,], [$, rewrite, in *simplest form*, expression $\big[[P \Rightarrow Q] \Rightarrow P\big]$ as _____. Ditto, rewrite $\big[P \Rightarrow [Q \Rightarrow P]\big]$ as _____.

15 15 b LBolt gives $G := \text{GCD}(413, 294) =$ _____. And $413S + 294T = G$, where $S =$ _____ & $T =$ _____ are integers.

15 15 c The number of permutations of "SETTEES", as a multinomial coefficient, is _____ (as a numeral) _____.

20 d As a single numeral, _____ is the following alternating sum:

$$*: \quad 1 - 3\cdot\binom{9}{1} + 9\cdot\binom{9}{2} - 27\cdot\binom{9}{3} + 81\cdot\binom{9}{4} - \ldots - 3^9\cdot\binom{9}{9}.$$

[*Hint:* First determine: Is the value positive, zero, or negative.]

5 10 40 **B2:** An integer-valued list $\mathcal{L} := (n_1, n_2, n_3, \ldots, n_9)$ is indexed by interval-of-integers $J := [1\,..\,9]$.

This J has _____ *non-void* subsets.

And J has _____ *non-void* subintervals. (Note: $[4\,..\,6]$ is a length-3 subinterval, and $[8\,..\,8]$ is a length-1 subinterval.)

Use PHP [Pigeon-hole Principle] to prove for each $\mathcal{L}$ as above that: *There exists a non-void set $\Omega \subset J$ of indices, st.*

$$\Big[\sum_{j\in\Omega} n_j\Big] \;\vdash\bullet\; 9\,.$$

You may use $\equiv$ for $\equiv_9$ i.e, congruence mod-9.

End of Class-B

OYOP: *In grammatical English* ***sentences****, write your essays on every* 2^{nd} *line* (usually)*, so I can easily write between the lines.* Please number the pages like "1 of 5", "2 of 5"... (or "1/5", "2/5"...)

B1:	___ ___ ___	100pts
B2:	___ ___	40pts
Total:	___ ___ ___	140pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: _____