B0: Show no work. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a Prof. King wears bifocals, and cannot read small handwriting. Circle one: **True!** **Yes!** **Who??**

b LBolt gives $G := \text{Gcd}(1533, 413) =$ _____ . And

$1533\,S + 413\,T = G$, where $S=$ _____ $\&\ T=$ _____ .

c⁺ Mod $K := 77$, the recipr. $\left\langle \frac{1}{15} \right\rangle_K =$ _____ $\in [0\mathbin{..} K)$.

[Hint: ♮] So $x=$ _____ $\in [0\mathbin{..} K)$ solves $4 - 15x \equiv_K 6$.

On $\mathbb{R}_+$, define several relations: Say that $x\mathcal{R}y$ **IFF** $y - x < 17$. Define $\mathcal{P}$ by: $x\mathcal{P}y$ **IFF** $x^{\log(y)} = 5$.

Say that $x\mathcal{I}y$ **IFF** $x + y$ is irrational.

Use $\bullet\mid$ for the "divides" relation on the positive integers: $k \bullet\mid n$ iff there exists a posint r with $rk = n$.

c₁ Please circle those of the following relations which are *transitive* (on their domain of defn).

$$\neq \qquad \bullet\mid \qquad \leqslant \qquad \mathcal{R} \qquad \mathcal{P} \qquad \mathcal{I}$$

c₂ Circle the *symmetric* relations:

$$\neq \qquad \bullet\mid \qquad \leqslant \qquad \mathcal{R} \qquad \mathcal{P} \qquad \mathcal{I}$$

c₃ Circle the *reflexive* relations:

$$\neq \qquad \bullet\mid \qquad \leqslant \qquad \mathcal{R} \qquad \mathcal{P} \qquad \mathcal{I}$$

d⁺ A pair (B, E) of *distinct* positive $\underline{ir}$rationals with B^E rational, is either $\left(\rule{1cm}{0pt}, \rule{1cm}{0pt} \right)$ or $\left(\rule{1cm}{0pt}, \rule{1cm}{0pt} \right)$.

Essay questions: For each question, carefully write a double– or triple–spaced, grammatical, essay solving the problem.

B1: Write, in fluid conventional English, the **contrapositive** of: "*If at least one of my hairs is gray, then no octogenarian is excused from class.*"

Write the **converse** of: "*If you don't cease lacking enthusiasm, then I won't stop avoiding double-negatives.*"

Write the **negation** of: "*All mimsy were the borogoves.*"

B2: Using set-builder notation, define the set of primes. $\text{PRIMES} = \big\{ n \in \text{WHAT} \mid \text{Conditions on } n \big\}$, using some of the symbols

such that, if, then, and, or, not, $0\,1\,2 \ldots$
$$\forall\ \exists\ \nexists \in \mathbb{N}\ \mathbb{Z}_+\ [a\mathbin{..}b)\ \bullet\ + =$$

and *avoiding* "*factor(s), divides, is-a-multiple, splits, irreducible, composite, Gcd, Lcm …*" and similar, uh, cheats. Every quantification must specify its set!

B3: **i** Carefully state the FTArithmetic. **ii** Use the FTArithmetic to carefully prove that $\sqrt{6}$ is irrational.

B4: Let $L(k) := [5^{[2k]}] - 1$. By induction on k, prove that
$$\boxed{\forall k \in \mathbb{N}: \quad L(k) \bullet\mid 3}.$$

B0:	___ ___	85pts
B1:	___ ___	45pts
B2:	___ ___	45pts
B3:	___ ___	45pts
B4:	___ ___	55pts
Total:	___ ___ ___	275pts

Print name _____ Ord: _____