Staple!

NT
MAS4203 *4D70* **Class-B** Prof. JLF King
Wedn., 25Jul2018

B4: *Show no work. Write* **DNE** *in a blank if the described object does not exist or if the indicated operation cannot be performed.*

a A *minimum* requirement for an LOR (letter-of-recommendation) from Prof. K is <u>two</u> courses. [Circle]:

Yes   **True**   **Darn tootin'***!*

b As 89 is prime, $\sigma_2(89^6) =$ _____ .
[Sum the geometric series, to write in closed form (no summation sign, nor …), but do not bother to compute powers.]

c Happily, $y=$ _____ is the smallest natnum satisfying $y \equiv_{60} -19$ and $y \equiv_{24} 5$.   [*Fusion.*]

d *Magic integers* G_1, G_2, G_3, each in $[0 .. 630)$, are such that the $g: \mathbb{Z}_{10} \times \mathbb{Z}_7 \times \mathbb{Z}_9 \to \mathbb{Z}_{630}$ mapping is a ring-isomorphism, where

$$g\big((z_1, z_2, z_3)\big) := \Big\langle z_1 G_1 + z_2 G_2 + z_3 G_3 \Big\rangle_{630} .$$

Then $G_3 =$ _____ $\in [0 .. 630)$.

e Integer $M := 23$ is prime. [Circle] each of the M-QRs, below. [*Hint:* At least two can be done by inspection.]

$$-1 \qquad 6 \qquad 7 \qquad 19$$

f Three Jacobi symbols: Two blanks are immed.:
$\left(\dfrac{4203}{2006}\right) =$ _____ . $\left(\dfrac{4203}{99}\right) =$ _____ . $\left(\dfrac{120}{27113}\right) =$ _____ .

g Let $N := 195$. Equation $N = [5 \cdot 4^3] - 5^3$ tells us that $\mathbf{u} =$ _____ $\in [0 .. N)$ satisfies $\mathbf{u}^3 \equiv_N 5$.

OYOP: *In grammatical English* **sentences***, write your essay on every* **third** *line* (usually), *so that I can easily write between the lines.*

B5: **i** State the Quadratic-reciprocity Thm.

ii For oddprimes $p \neq 5$: *There exist* $b, c \in [1 .. p)$ *satisfying congruence* $b^2 + 5c^2 \equiv_p 0$ IFF non-negative residue
$\langle p \rangle_{20} \in \Big\{$ _____ $\Big\}.$

iii Use the Quad-Reciprocity Theorem to prove your result in (ii). [*Hint:* Use complete, grammatical, correctly punctuated **sentences**, please.]

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│   End of Class-B   │
└─────────────────────────┘

B4: __ __ __   120pts

B5:   __ __   75pts

Total: __ __ __   195pts

Please PRINT your name *and* ordinal*. Ta:*

Ord:
....

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature:
....