Linear Algebra
MAS3114 3164

Class-B

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Hello. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than $.9797\cdots$.

B1: Show no work.

z Prof. King wears bifocals, and cannot read small handwriting. Circle one: **True!** **Yes!** ***Who?***

a The matrix-product $\begin{bmatrix} 2 & 1 & 4 \\ 0 & -1 & 1 \\ -2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{2} & 0 & \sqrt{2} \\ 0 & -1 & 5 \end{bmatrix}$ equals

b Consider these two matrices:

$$\mathsf{R} := \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \quad \text{and} \quad \mathsf{A} := \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}.$$

Product matrix $[RA]^{40}$ equals

[*Hint:* You don't need multiply matrices. Geometrically, what motion do these matrices represent.]

c Suppose T is a linear map from $\mathbb{R}^3$ to $\mathbb{R}^2$. Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis for $\mathbb{R}^3$, and let $\{\mathbf{v}_1, \mathbf{v}_2\}$ be the standard basis for $\mathbb{R}^2$. Suppose that $T(\mathbf{e}_1) = 17\mathbf{v}_1 - 2\mathbf{v}_2$ and $T(\mathbf{e}_2) = 6\mathbf{v}_2$ and $T(\mathbf{e}_3) = -4\mathbf{v}_1 - 3\mathbf{v}_2$.

Then the matrix of T is: .

d Over $\mathbb{Q}$, the inverse of $\mathsf{E} := \begin{bmatrix} 1 & x & z \\ & 1 & y \\ & & 1 \end{bmatrix}$ is

e Let $\mathsf{K} := \left[\begin{smallmatrix} 2 & 1 \\ 5 & 3 \end{smallmatrix}\right]$. The minimum-deg **monic** polynomial $h()$ st. $h(\mathsf{K}) = \left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right]$ is $h(x)=$.

f Let $\mathbf{v}_1 := \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 := \begin{bmatrix} 2 \\ 1 \\ -2 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 := \begin{bmatrix} 4 \\ W \\ 3 \\ Y \end{bmatrix}$, So $\mathbf{v}_3 \in \mathrm{Spn}(\mathbf{v}_1, \mathbf{v}_2)$ when $W=$ & $Y=$.

And $\mathbf{v}_3 = \alpha\mathbf{v}_1 + \beta\mathbf{v}_2$, where $\alpha=$ and $\beta=$.

g The 3×3 elem-matrix whose lefthand action adds 8 times row-2 to row-1 is .

h An example of 2×2-matrices with $\mathsf{A}^2 \neq \mathsf{B}^2$, yet with $\mathsf{A}^3 = \mathsf{B}^3$, is $\mathsf{A} =$ and $\mathsf{B} =$.

Essay question: On your own sheets of paper, write a soln using complete sentences, explaining a bit about HOW this problem is solved.

B2: A system of 3 linear equations in unknowns $x_1, \ldots, x_5$ reduces to the augmented matrix

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 1 & 12 \\ 0 & 0 & 1 & 0 & -8 & 34 \\ 0 & 0 & 0 & 1 & 5 & -56 \end{array}\right],$$

which is *almost* in RREF. Please circle each *pivot*.

OYOP, describe the *general solution* in this form,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \alpha \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \beta \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \gamma \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \ldots$$

where each $\alpha, \beta, \gamma, \delta, \ldots$ is a free variable (either x_1 or...or x_5), and each column vector has specific numbers in it. $\mathrm{Dim}(SolnFlat) =$.

End of Class-B

B1:	___ ___ ___	180pts
B2:	___ ___	45pts
Total:	___ ___ ___	225pts

Print name Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: