

B1: Show no work. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\}$ $\neq 0$.

a Perm $\pi := [4, 5, 6, 7, 8, 1, 2, 3]$ has $\text{Sgn}(\pi) = +\mathbf{1} \ -\mathbf{1}$.

b Every 5×5 matrix M has $\text{Det}(7M) = \alpha \cdot \text{Det}(M)$, where α is circle
 $7^{25} \quad 7^5 \quad 7^4 \quad 7 \quad 5^7 \quad 25^7 \quad 5^{49} \quad \text{None-of-these}$

c Inverse of $\begin{bmatrix} 1 & & \\ 4 & -1 & \\ 5 & 6 & 2 \end{bmatrix}$ is $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$.

d Determinant of $M := \begin{bmatrix} -5 & 4 & 0 \\ 1 & 3 & -2 \\ 2 & 1 & 0 \end{bmatrix}$ is .

The characteristic-poly of M is $Ax^3 + Bx^2 + Cx + D$, where $B = \text{.....}$ and $C = \text{.....}$.

e Mats $U = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}, V = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

are \mathbb{R} -matrices such that $U^2 \neq V^2$, yet $U^3 = V^3$.

f Let $R(x) := \begin{bmatrix} 3x^2 & 7x^5 - 8 & 6x \\ 5 & 9x & 2x - 1 \\ 8x + 4 & x^6 + 1 & x^4 + 2 \end{bmatrix}$.

The high-order term of polynomial $\text{Det}(R(x))$ is Cx^N , where $C = \text{.....}$ and $N = \text{.....}$.

g Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by reflecting the plane across the θ -angle line. [Picture on blackboard.] Then $M := \llbracket T \rrbracket_{\mathcal{E}}^{\mathcal{E}}$ equals $\begin{bmatrix} \cos(\alpha) & \cos(\beta) \\ \sin(\alpha) & \sin(\beta) \end{bmatrix}$, where $\alpha = \text{.....}$ and $\beta = \text{.....}$. Also, $\text{Det}(M) = \text{.....}$.

OYOP: Essay: *Write on every **third** line, so that I can easily write between the lines. In grammatical English sentences, prove the following:*

B2: Here, each matrix is a 2×2 matrix. Write $C \looparrowright D$, “matrix C **loop-arrows** D ”, if *there exists* an invertible U such that $UCU^{-1} = D$.

x What are the properties that that \looparrowright needs to have, to be an “equivalence relation”?

Prove that \looparrowright is an equivalence relation.

y Prove: *If at least one of S, T is invertible then $ST \looparrowright TS$.*

z Matrices $A = \begin{bmatrix} & & \\ & & \end{bmatrix}, B = \begin{bmatrix} & & \\ & & \end{bmatrix}$

are such that $AB \not\looparrowright BA$, *with proof*.

End of Class-B

B1: 160pts

B2: 80pts

Total: 240pts