

Abstract Algebra
MAS4301 09B1

Class-B

Prof. JLF King
Wednesday, 30Oct2019

Hi. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\}$ $\neq 0 \neq$ *Empty-word*.

B4: Show no work.

a Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$500 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is Monday, March 23, 2020.

Circle: **Yes** **True** **Résumé material!**

b Let $V_K := G \times \mathbb{Z}_K \times G$, where $G := (\mathbb{Z}_2, +)$. As A Decreasing Product of Integers, $|\text{Aut}(V_6)| =$ _____.

c $\Pi\Pi$ -auts **T, S, R, F** [Traffic-light, Swizzle, Rotation, Flip] generate the group Γ of 4×4 -auts. Let Ω be the set of 10 $\Pi\Pi$ s on which Γ acts, with **V** the leftmost *Vertical* $\Pi\Pi$, and **D** the upper-left to lower-right *Diagonal* $\Pi\Pi$. The Orb-Stab thm implies $|\text{Stab}_\Gamma(\mathbf{V})| =$ _____ and $|\text{Stab}_\Gamma(\mathbf{D})| =$ _____. In std form, $\text{Stab}_\Gamma(\mathbf{V}) = \left\{ \text{_____} \right\}$.

d *Shuffling* $2N$ -card deck: Put the **upper** N in your **right hand**, and the **lower** N in **left hand**. Drop a **RH** card, then a **LH**, then **RH**, etc. [New bottom-card came from **RH**; new top-card from **LH**.] So $S_N := \text{Sign}(\pi_N) =$ _____. And S_{2019} is circle **+1** **-1** . _____

e Mod $K := 51$, the recipr. $\langle \frac{1}{20} \rangle_K =$ _____ $\in [0 .. K)$.
[Hint: $\frac{1}{2}$] So $x =$ _____ $\in [0 .. K)$ solves $5 - 20x \equiv_K 2$.

OYOP: In grammatical English *sentences*, write your essay on every 2nd line (usually), so that I can easily write between the lines.

B5: Suppose τ is the unique element of order 2 in a [possibly infinite] group G . Prove that $\tau \in Z(G)$, the center of G .

B-Home: ___ ___ ___ 325pts

B4: ___ ___ ___ 120pts

B5: ___ ___ 45pts

Total: ___ ___ ___ 490pts