Abstract Algebra
MAS4301 3175 **Class-B** Prof. JLF King
17Feb2008

Hello. Write **DNE** if the object does not exist or the operation cannot be performed. NB: $\mathbf{DNE} \neq \{\} \neq 0 \neq$ *Empty-word.*.

Let $\mathtt{F}$ and $\mathtt{R}$ be the *flip* and *rotation* in the dihedral group $\mathbb{D}_N$, with $\mathtt{F}^2 = \mathbf{e}$, $\mathtt{R}^N = \mathbf{e}$ and $\mathtt{RFRF} = \mathbf{e}$. Use $\mathtt{R}^j$ and $\mathtt{R}^j\mathtt{F}$ as the standard form of each element in $\mathbb{D}_N$.

Use $\mathbb{Y}_N$ or $\mathbb{Y}(N)$ to denote the cyclic group of order N.

Fill-in *all* blanks on this sheet **including** the blanks for the essay questions!

B4: Show no work.

a
Euler $\varphi(1,000,000) =$ ....
Express your answer as a product $p_1{}^{e_1} \cdot p_2{}^{e_2} \cdot \ldots$ of *primes* to posint powers, with $p_1 < p_2 < \ldots$.

b
Mod $K := 3175$, the recipr. $\left\langle \frac{1}{73} \right\rangle_K =$ .... $\in [0 .. K)$.
[*Hint: ♮*] So $x =$ .... $\in [0 .. K)$ solves $4 - 73x \equiv_K 1$.

c
In $\mathbb{S}_{15}$, in terms of multinomial-coeffs and factorials: There are .... many solo 15-cycles. And the
$\#\left\{ \begin{matrix} \text{Elements of} \\ \text{order } 35 \end{matrix} \right\} =$ ....

d
In $\mathbb{S}_{19}$ there are ..... many signatures whose elts have order 15. They are: ....
.
....

e
Cards $0, 1, \ldots, 7$ are fed into a shuffling machine, then the output is fed back in, resulting in $5, 4, 0, 2, 6, 7, 1, 3$. So after the first pass, the cards were in order

....

f
Each $h \in \mathbb{D}_{10}$ yields an inner-auto $J_h(x) := hxh^{-1}$. Writing elts in form $\mathtt{R}^k\,\mathtt{F}^s$, two *distinct* $\alpha, \beta \in \mathbb{D}_{10}$ with
$J_\alpha = J_\beta$ are $\alpha =$ .... and $\beta =$ ....

g
$\#\mathrm{Inn}(\mathbb{D}_{10}) =$ .... and $\#\mathrm{Aut}(\mathbb{D}_{10}) =$ ....

Essay questions: **Fill-in all blanks.** *For each question, carefully type a double– or triple–spaced essay solving the problem. Each essay starts a new page.*

B5: Given an example of two perms $\alpha, \beta \in \mathbb{S}_\mathbb{Z}$, with $\mathrm{Ord}(\alpha) = 2 = \mathrm{Ord}(\beta)$, yet $\mathrm{Ord}(\alpha\beta) = \infty$. What is the cycle-signature of your $\alpha\beta$?

B6: Use a full page to draw the lattice of subgroups of $\mathbb{S}_3$. Describe the subgps by defining specific elts in cycle notation, then showing the subgps in form, e.g, $\langle z, y \rangle$.

End of Class-B

B-Home: ___ ___ ___ 265pts

B4: ___ ___ 95pts

B5: ___ ___ 15pts

B6: ___ ___ 20pts

Total: ___ ___ ___ 395pts

Ord:
....