Advanced Calc
MAA4102 & MAA5104

Class-B

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Please. Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$.Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!)

Abbrevs: **ITOf** for "in terms of". **st.** for "such that" **seq** for "sequence" **posint** for "positive integer" **DL** for "Dedekind-Left", the lefthand atom of a Dedekind cut. **poly** for "polynomial" **coeff** for "coefficient"

For each of the limit questions, write "$+\infty$", "$-\infty$", a real number, or *–if none of these–* "DNE". Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797$\cdots$.

B0: For the following Greek letters, please write the **other case**, along with their names.
Eg: "α: ⌞.....⌟ B: ⌞.....⌟." You fill in: A *alpha* *beta*.
Ω: ⌞.....⌟ Δ: ⌞.....⌟ N: ⌞.....⌟
ξ: ⌞.....⌟ ρ: ⌞.....⌟ λ: ⌞.....⌟

B1: Show no work.

z Prof. King thinks that every **if**-word should be matched by an explicit **then**-word, such as "*then*", "*necessarily*" or "*we have that*". Circle one:
True **Oui** **Da** **Yes** *Uh, Did I miss something??*

a $\lim_{n\to\infty} \frac{5n+[7+4n][1-n]}{4+n^2}=$ ⌞.....⌟.

b (Right/left hand limits): $\lim_{x\to 3^+} \frac{|3-x|}{3-x}=$ ⌞.....⌟.
$\lim_{x\to 3^-} \frac{|3-x|}{3-x}=$ ⌞.....⌟.

c The set $S\subset\mathbb{R}$ is upper-bounded, yet has no LUB (in $\mathbb{R}$). So $S=$ ⌞.....⌟.
A particular upper-bnd for your S is ⌞.....⌟ $\in\mathbb{R}$.

d For $r\in[-\infty,+\infty]$, its DL is $\{q\in\mathbb{Q}\mid q<r\}$, written $\mathbf{L}_r$.
Given $s,t\in\mathbb{R}$, define "$s+t$" by defining
$\mathbf{L}_{s+t}\coloneqq$ ⌞.....⌟
ITOf $\mathbf{L}_s$ and $\mathbf{L}_t$ and addition-of-rationals.
If you apply this defn to $s\coloneqq-\infty$ and $t\coloneqq+\infty$, the definition says that $-\infty+[+\infty]=$ ⌞.....⌟.

e For a set Ω of extended reals, define its supremum $r\coloneqq\sup(\Omega)$ by defining its DL:
$\mathbf{L}_r\coloneqq$ ⌞.....⌟.

f Use $\lfloor\cdot\rfloor$ for the "floor fnc" (the "greatest integer fnc"). So $\lfloor\pi\rfloor=$ ⌞.....⌟ and $\lfloor-\pi\rfloor=$ ⌞.....⌟.

g Let E be the set $\left\{5+\left[[-1]^n\cdot\frac{2n-1}{n}\right]\right\}_{n\in\mathbb{Z}_+}$. Then $\sup(E)=$ ⌞.....⌟ and $\inf(E)=$ ⌞.....⌟.

h $\lim_{x\to 0}\frac{\sqrt{25+x}-\sqrt{25-x}}{x}=$ ⌞.....⌟.

i Let $F(x)\coloneqq x^{\sin(x)}$. Its derivative, then, is
$F'(x)=$ ⌞.....⌟.
[*Hint:* How is $x^{\sin(x)}$ defined ITOf the exponential fnc?]

Essays. On your own sheets of lined paper, give the following definitions or proofs. No "scratch work" accepted, only complete, grammatical, coherent sentences. Write **every 2nd or every 3rd line** for math essays.

B2: I Write the *definition* of "$\lim_{x\to 7} x^2=4$". [*Hint:* You need 3 quantifiers, an ε and a δ.]

II Given seqs. with $\lim(\vec{\mathbf{b}})=2$ and $\lim(\vec{\mathbf{c}})=4$, let $\vec{\mathbf{s}}\coloneqq\vec{\mathbf{b}}+\vec{\mathbf{c}}$. Given $\varepsilon>0$, produce –with careful proof– a posint N st.: $\forall k\geqslant N:\ |s_k-6|<\varepsilon$.

B3: State the Fundamental Thm of Arithmetic.
Use the FTArithmetic to give a careful proof that there are *no* posints N,D for which $\left[\frac{N}{D}\right]^3=4$.

End of Class-B

B0:	___ ___	18pts
B1:	___ ___ ___	200pts
B2:	___ ___	75pts
B3:	___ ___	75pts

KnowsOrd: ___ 2pts

Total: ___ ___ ___ 370pts

Print name Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: