

**ACTroids.** Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

All M<sub>S</sub>es, here, are subspaces of  $\mathbb{R}$ .

**B4:** Show no work.

**[z<sub>5</sub>]** The author of our text is **Circle**: **Archimedes**  
**Bubba** **Buck** **Doker** **Euler** **Machen** **Rosenlicht**

**[a<sub>10</sub>]** Repeating decimal  $0.7\overline{20}$  equals  $\frac{n}{d}$ , where posints  
 $n \perp d$  are  $n =$  \_\_\_\_\_ and  $d =$  \_\_\_\_\_.

**[b<sub>10</sub>]** Define  $X :=$  \_\_\_\_\_  $\subset \mathbb{R}$  st. the  $X$ -open  
ball  $B := X\text{-Bal}_3(0) =$  \_\_\_\_\_ satisfies  
 $B \subsetneq \text{Cl}_X(B) =$  \_\_\_\_\_  $\subsetneq X\text{-CldBal}_3(0) =$  \_\_\_\_\_

**[c<sub>10</sub>]** With  $\alpha(\cdot, \cdot)$  the arctan metric on  $\mathbb{R}$ , the  
 $\alpha\text{-Diam}(\text{PRIMES}) =$  \_\_\_\_\_.  
[Hint: No  $\alpha()$  should appear in your ans. But arctan() can.]

**[d<sub>15</sub>]** Sets  $A :=$  \_\_\_\_\_ and  $B :=$  \_\_\_\_\_ have  
 $\partial_{\mathbb{R}}(A) =$  \_\_\_\_\_ and  $\partial_{\mathbb{R}}(B) =$  \_\_\_\_\_. Moreover,  
 $= \partial_{\mathbb{R}}(A) \cap \partial_{\mathbb{R}}(B) \subsetneq \partial_{\mathbb{R}}(A \cap B) =$  \_\_\_\_\_.

**[e<sub>15</sub>]** Sets  $C :=$  \_\_\_\_\_ and  $D :=$  \_\_\_\_\_ have  
 $\partial_{\mathbb{R}}(C) =$  \_\_\_\_\_ and  $\partial_{\mathbb{R}}(D) =$  \_\_\_\_\_. Further,  
 $= \partial_{\mathbb{R}}(C) \cap \partial_{\mathbb{R}}(D) \supsetneq \partial_{\mathbb{R}}(C \cap D) =$  \_\_\_\_\_.

**Essay question:**

**B5:** In  $\mathbb{R}$ : Prove, for all sets  $E_1, E_2 \subset \mathbb{R}$ , that

$$1: \quad \partial(E_1) \cup \partial(E_2) \supset \partial(E_1 \cap E_2).$$

[Hint: Fixing a point  $q \in \partial(E_1 \cap E_2)$ , we know there exist sequences  $\vec{b} \subset E_1 \cap E_2$  and  $\vec{x} \subset [E_1 \cap E_2]^c$  converging to  $q$ . You need to show, either for  $j=1$  or  $j=2$ , that  $E_j^c$  includes a sequence  $\vec{y}$  that converges to  $q$ . Also, explain why the existance of such a  $\vec{y}$  is sufficient to establish (??).]

**HONOR CODE:** *"I have neither requested nor received help on this exam other than from my professor."*

Signature: \_\_\_\_\_