ACES
MAA4102, MAA5104

Prac-B

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17Feb2008

Hello. Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

Fill-in *all* blanks on this sheet **including** the blanks for the essay questions!

B1: Show no work.

a The point $P := (5,-1)$, in the plane, has orthogonal proj. $\text{Proj}(P) = ($ ____ , ____ $)$ on the line $y = 1 + 3x$.

b In $\mathbb{R}^3$, the point $P := (2,-1,3)$ has orthogonal projection $\text{Proj}(P)=($ ____ , ____ , ____ $)$ on the line passing through $(2,2,4)$ and the origin.

c In $\mathbb{R}$, open intervals $J_n := ($ ____ , ____ $)$ intersect to a **non**-open set $\bigcap_{n=1}^{\infty} J_n =$ ____________ .

d Let $S := \{q \in \mathbb{Q} \mid 3 < q \leqslant 5\} = \mathbb{Q} \cap (3,5]$. Then:
$\text{Cl}_{\mathbb{R}}(S)=$ ________ . $\text{Itr}_{\mathbb{R}}(S)=$ ________ .
$\text{Cl}_{\mathbb{Q}}(S)=$ ________ . $\text{Itr}_{\mathbb{Q}}(S)=$ ________ .

e Let $S := \{q \in \mathbb{Q} \mid 3 < q \leqslant 5\} = \mathbb{Q} \cap (3,5]$. Then:
$\text{Bdry}_{\mathbb{R}}(S)=$ ________ . $\text{Bdry}_{\mathbb{Q}}(S)=$ ________ .

f On $\mathbb{Z}_+$, write $x \,\$\, y$ IFF $xy < 0$. So \$ is *Circle*
Transitive *T F*. **Symmetric** *T F*.
Reflexive *T F*.

On $\mathbb{Z}$, say that $x \nabla y$ IFF $x - y \leqslant 1$. Then ∇ is:
Trans. *T F*. **Symm.** *T F*. **Reflex.** *T F*.
(Be *careful* on <u>both</u> parts!)

On $\mathbb{R}_+$, define several relations: Say that $x\mathcal{R}y$ IFF $y - x < 17$. Define $\mathcal{P}$ by: $x\mathcal{P}y$ IFF $x^{\log(y)} = 5$.
Say that $x\mathcal{I}y$ IFF $x + y$ is <u>ir</u>rational.
Use $\bullet\!|$ for the "divides" relation on the positive integers: $k \bullet\!| \, n$ iff there exists a posint r with $rk = n$.

f_1 Please circle those of the following relations that are *transitive* (on their domain of defn).

$\neq \quad \bullet\!| \quad \leqslant \quad \mathcal{R} \quad \mathcal{P} \quad \mathcal{I}$

f_2 Circle the *symmetric* relations:

$\neq \quad \bullet\!| \quad \leqslant \quad \mathcal{R} \quad \mathcal{P} \quad \mathcal{I}$

f_3 Circle the *reflexive* relations:

$\neq \quad \bullet\!| \quad \leqslant \quad \mathcal{R} \quad \mathcal{P} \quad \mathcal{I}$

Essay questions: **Fill-in all blanks.** *For each question, carefully write a triple–spaced essay solving the problem.*

B2: Define: "On a set E, a binary relation ∇ is an ***equivalence relation*** IFF…". *Make sure* to <u>define</u> any terms like "<u>reflexive</u>" that you use in your defn.!

Let $\mathbf{P}$ be the set of ordered integer-pairs (n, d), with $d \neq 0$. Define relation \$ on $\mathbf{P}$ by

$$(N, D) \,\$\, (x, y) \qquad \text{IFF} \qquad N \cdot y = x \cdot D.$$

Prove, in detail, that \$ is an equivalence relation.

B3: A ***field*** $(F, +, 0, \cdot, 1)$ satisfies these axioms:

B4: "MS (Ω, d) is ***connected***" means that…

B5: Sets U_i are open in MS (Ω, d). i Prove that $U_1 \cup U_2$ is Ω-open.

ii Prove that $U_1 \cap U_2$ is Ω-open.

B6: In MS (Ω, d), sequence $\vec{b} \subset \Omega$ converges to both q and r in Ω. Prove that $q = r$, by showing that $\mathsf{d}(q, r) = 0$. [*Hint:* Use the Triangle Inequality.]

B7: Below, $\mathbf{V}, \mathbf{W}, \mathbf{E}$ are real vectorspaces.

a A map $f\colon \mathbf{V} \times \mathbf{W} \to \mathbf{E}$ is ***bilinear*** if…

b A map $\langle \cdot, \cdot \rangle$ from $\mathbf{V} \times \mathbf{V} \to \mathbb{R}$ is an ***inner product*** if…

B8: Use $\langle \cdot, \cdot \rangle$ for an inner-product on $\mathbb{R}$-vectorspace $\mathbf{V}$.

x1 <u>State</u> the Cauchy-Schwarz Inequality Thm, carefully stating the IFF-condition for equality. x2 **<u>Prove</u>** the C-S Inequality Thm, using the axioms for inner-product.

B9: Our space is $\mathbb{R}$ with the usual Euclidean metric $\mathsf{d}(x, z) := |x - z|$. I These *closed* bnded non-void intervals $A_n :=$ ____________ , when unioned, form a set $\bigcup_{n=1}^{\infty} A_n =$ ____________ which is not closed.

II Suppose that $U, V_1, V_2, \ldots$ are open sets of $\mathbb{R}$, and $E, K_1, K_2, \ldots$ are closed sets. **Circle** those of the following sets which are <u>guaranteed</u> to be *closed* in $\mathbb{R}$.

$E \smallsetminus U. \qquad U \smallsetminus E. \qquad K_1 \smallsetminus E. \qquad \bigcap_{n=1}^{\infty} K_n.$

$\mathbb{R} \smallsetminus \left[\bigcup_{n=1}^{\infty} V_n\right]. \qquad E \cup K_1. \qquad E \cap K_1.$