Sets and Logic MHF3202 7860	**Class-B**	Prof. JLF King Friday, 22Mar2024

B4: Short answer. Show no work. Write LARGE.

Write **DNE** <u>if</u> the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

a For a LOR (letter-of-recommendation), Prof. K requires <u>two</u> courses, or a Special Topics [e.g, my NUMBER THEORY AND CRYPTOGRAPHY], or graduate course [Circle]:

Yes   **True**   **Darn tootin'!**

b The ***Threeish-numbers*** comprise $\mathcal{T} := 1 + 3\mathbb{N}$.

$\mathcal{T}$-number $385 \stackrel{\text{note}}{==} 35 \cdot 11$ is $\mathcal{T}$-*irreducible*:   T   F

Threeish $N := 85$ is **<u>not</u>** $\mathcal{T}$-*prime* because $\mathcal{T}$-numbers $J :=$ _____ and $K :=$ _____ satisfy

that $N \bullet\!\mid [J \cdot K]$, **yet** $N \nmid J$ and $N \nmid K$.

Also, $\mathcal{T}$-GCD$(175, 70) =$ _____ .

c On a K-elt set Ω, the number $\#_K$ of **reflexive symmetric** binrels is _____ .

In particular, $\#_5 =$ _____ .

d On a 3-set, there are _____ many equiv.relations.

e Multinomial coefficient $\binom{9}{4, 2, 3} =$ _____ $=$ _____ .

[Write your answer as a product of binomial coeffs, then compute the product as a single integer,]

B5: Consider a strict well-order $\prec$ on set **U**, and a strict well-order $<$ on $\boldsymbol{\Gamma}$. Define binrel $\ll$ on $\mathbf{U} \times \boldsymbol{\Gamma}$ by:

$$(b, \alpha) \ll (c, \beta)$$

IFF   *Either* $b \prec c$ *or* $[b = c$ <u>and</u> $\alpha < \beta]$.

Prove: *Relation* $\ll$ *is a well-order on* $\mathbf{U} \times \boldsymbol{\Gamma}$.

[*You may assume that* $\ll$ *is a total-order.*]

B6: Define: "On a set E, a binary relation ∇ is an **equivalence relation** IFF...". *Make sure* to <u>define</u> any terms like "<u>reflexive</u>" that you use in your defn.!

Let $\mathbb{P}$ be the set of ordered integer-pairs (n, d), with $d \neq 0$. Define relation $\boldsymbol{C}$ on $\mathbb{P}$ by

$$(N, D) \, \boldsymbol{C} \, (x, y) \qquad \text{IFF} \qquad N \cdot y = x \cdot D.$$

Prove, in detail, that $\boldsymbol{C}$ is an equivalence relation.

B4:	___ ___	95pts
B5:	___ ___	45pts
B6:	___ ___	45pts
Total:	___ ___ ___	185pts

NAME: ....

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....