Linear Algebra
MAS4105 5441

PB-Practice

Prof. JLF King
Touch: 16Mar2016

Recall that InClass-B will be on **04Nov, Friday**.

Please. Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797$\cdots$. Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!)

PB0: Short answer: Show no work. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a Binomial coefficient $\binom{7}{2}=$ ⌞ . . . . . ⌟.
Multinomial coeff $\binom{9}{3,4,2}=$ ⌞ . . . . . ⌟.

b Let $\mathsf{B} := \begin{bmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$. [*Hint:* Write B as a diag-matrix plus a nilpotent matrix.]

Then $\mathsf{B}^{2005} =$ ⌞ . . . . . ⌟.

c $\mathsf{C} := \begin{bmatrix} 2 & 0 & -3 \\ 2 & -1 & 5 \\ 3 & 0 & 1 \end{bmatrix}$. Matrix $P_{\mathsf{C}}(2)=$ ⌞ . . . . . ⌟.
And the char.poly is $p_{\mathsf{C}}(x)=$ ⌞ . . . . . ⌟.

d Greek *stokhos* means ⌞ . . . . . ⌟.

Below, let *AT* mean "Always True", *AF* mean "Always False" and *Nei* mean "Neither always true nor always false".

1 If 3 is an M-eval, then 9 is an M^2-eval. *AT AF Nei*

2 If 9 is an M^2-eval then 3 is an M-eval. *AT AF Nei*

3 Suppose $\mathbf{w}\neq\mathbf{0}$ is an evec for M^2. Then $\mathbf{w}$ is an evec for M. *AT AF Nei*

PB1: State and prove Cramer's Thm. Let $\mathsf{E} := \begin{bmatrix} x & y & z \\ 7 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$. Let $h(x,y,z)$ be the $(3,1)$-entry of E^{-1}. Then $h(x,y,z)=$ ⌞ . . . . . ⌟, a rational fnc.

PB2: Diagonalize the 3×3 matrix K, on the blackboard, and compute K^{4105}.

PB3: Matrix $\mathsf{M} = \begin{bmatrix} \mathsf{A} & \mathsf{B} \\ \mathsf{C} & \mathsf{D} \end{bmatrix}$, where A and D are 5×5 and 7×7, resp. Suppose C is the 7×5 zero-matrix. Prove that $\mathrm{Det}(\mathsf{M}) = \mathrm{Det}(\mathsf{A}) \cdot \mathrm{Det}(\mathsf{D})$.

PB4: Below, $\mathsf{G} = \begin{bmatrix} \mathsf{A} & \mathsf{B} \\ \mathsf{C} & \mathsf{D} \end{bmatrix}$, with $\mathsf{A}, \mathsf{B}, \mathsf{C}, \mathsf{D}$ each 2×2. Give an *example* where

$$\mathrm{Det}(\mathsf{G}) \quad \neq \quad \mathrm{Det}(\mathsf{A}) \cdot \mathrm{Det}(\mathsf{D}) \; - \; \mathrm{Det}(\mathsf{B}) \cdot \mathrm{Det}(\mathsf{C}) \,.$$

Now produce an example where

$$\mathrm{Det}(\mathsf{G}) \quad \neq \quad \mathrm{Det}(\mathsf{AD} \; - \; \mathsf{BC}) \,.$$

PB5: Poly $h(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3$ satisfies that $h(2)=0$, $h(3)=1$, $h(5)=0$, $h(6)=2$. Use **Lagrange Interpolation** to compute $b_0=$ ⌞ . . . . . ⌟; $b_1=$ ⌞ . . . . . ⌟; $b_2=$ ⌞ . . . . . ⌟; $b_3=$ ⌞ . . . . . ⌟. Show the details.

PB6: OASSOP, write out the following sentences, and complete them to give the correct definitions. Be specific with phrases "every", "some", "there exists", etc.. Define "trivial soln" before using it. Let $\mathbf{V} := \mathbb{C}^{\times 5}$. All matrices below are 5×5 complex matrices unless otherwise stated.

A col-vector $\mathbf{v} \in \mathbf{V}$ *is an* ***eigenvector*** *for* M *if...*
Collection $\mathcal{C} := \{\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_K\}$ *of* $\mathbf{V}$*-subspaces is* ***linearly independent*** *IFF ...*
Matrix M *is* ***nilpotent*** *if ...*
Matrix M *is an* ***orthogonal matrix*** *if ...*
Matrix M *is* ***unitary*** *if ...*
Fix $\beta \in \mathbb{C}$. *The* M***-algebraic multiplicity*** *of* β *is ....* *The* M***-geometric multiplicity*** *of* β *is ....*
Matrices B, E *are* ***similar*** *if ....* *They are* ***row equivalent*** *if ....*
An $K \times N$ *matrix* R *is in* ***reduced row echelon form*** *IFF ...* (Define pivot col/row and free col.)

The ***degree*** *of poly* $f(x,y,z)$ *is ....* *This poly* f *is* ***homogeneous*** *IFF ....*
Degree-5 poly $g(x)$ ***splits*** *over* $\mathbf{F}$ *IFF ...*
A field $\mathbf{F}$ *is* ***algebraically closed*** *if ...*

PB7: Produce, with proof, a $\mathbb{Q}$-matrix whose char-poly is $x^4 + 3$. State the Cayley-Hamilton Thm.

Please PRINT your name and ordinal. Ta:

Ord:

Filename: Classwork/LinearAlg/LinA2005t/b-cl-PRAC.LinA2005t.latex
As of: *Monday 31Aug2015.* Typeset: *16Mar2016 at 02:19.*