Staple!

Ord: ___ ___

Sets and Logic
MHF3202

SeLo-makeup-B

Prof. JLF King
Touch: 30Sep2019

B1:	___ ___	45pts
B2:	___ ___	45pts
B3:	___ ___ ___	105pts
Total:	___ ___ ___	195pts

Print name ⌞. . . . . ⌟ Ord: ⌞. . . . . ⌟

B1: *Essay, on your own paper, triple-spaced:* Please prove: THM: *For each posint K there exists $N \in \mathbb{Z}_+$ st. each number $N+1, N+2, \ldots, N+K$ is composite.*

B2: *Essay, on your own paper, triple-spaced:* Please prove: THM: *For each posint L, if $M_L := 2^L - 1$ is prime, then L is prime.* [*Hint:* Contrapositive. Suppose $L = b \cdot c$ for some $b,c \in [2\,..\,\infty)$. Construct a factorization of $M_L = \beta \cdot \gamma$ which is **non-trivial**. Don't forget to show that $\beta,\gamma \neq 1$, and both are positive. Oh, and that β and γ are *integers*.]

B3: Show no work.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0 \neq$ *Empty-word*.

___ a Using *only* symbols $\boldsymbol{H}, \boldsymbol{D}, \wedge, \vee, \neg, T, F,], [$, rewrite (in simplest form) expression $C \Rightarrow [[\neg B] \Rightarrow C]$ as ⌞. . . . . ⌟. Ditto, rewrite $[B \Rightarrow C] \Rightarrow C$ as ⌞. . . . . ⌟.

___ b Rewrite $y \in \left[G \cap [\bigcup_{k \in B} H_k] \right]$ without $\cup, \cap$, by using only $\boxed{\exists \text{ st. } \forall : \quad \in \ni \ \vee \ \wedge \ ] \ [\ x \ F \ G \ k \ B}$, as ⌞. . . . . ⌟.

___ c LBolt: GCD$(70, 42)$= ⌞. . . . . ⌟ $\cdot 70 +$ ⌞. . . . . ⌟ $\cdot 42$.
So (LBolt again) $G := \text{GCD}(70, 42, 35)$= ⌞. . . . . ⌟ and
⌞. . . . . ⌟ $\cdot 70 +$ ⌞. . . . . ⌟ $\cdot 42 +$ ⌞. . . . . ⌟ $\cdot 35 = G$.

___ d+ Let B:=132, M:=112, T:=12. Congruence $B \cdot x \equiv_M T$ has *reduced congruence* $\beta \cdot y \equiv_\mu \tau$, where β= ⌞. . . . . ⌟, μ= ⌞. . . . . ⌟, τ= ⌞. . . . . ⌟, and soln y= ⌞. . . . . ⌟ $\in [0\,..\,\mu)$.
Congruence $B \cdot x \equiv_M T$, has ⌞. . . . . ⌟ many solns, which are x= ⌞. . . . . ⌟ $\in [0\,..\,M)$.

___ e $\boldsymbol{\tau}([45]^7)$= ⌞. . . . . ⌟ and $\boldsymbol{\sigma}([45]^7)$= ⌞. . . . . ⌟.
And Euler $\boldsymbol{\varphi}(55 \cdot 12100)$ = ⌞. . . . . ⌟.
Express your answer as a product ${p_1}^{e_1} \cdot {p_2}^{e_2} \cdot \ldots$ of *primes* to posint powers, with $p_1 < p_2 < \ldots$.

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ⌞. . . . . ⌟