



Staple!

Combinatorics
MAD4203 3214

Class-B

Prof. JLF King
Wednesday, 18Oct2017

Ord: _____

Welcome. Write DNE in a blank if the described object does not exist or if the indicated operation cannot be performed.

Use $\mathcal{S}(N, K)$ for 2Stirling #s, and use $\mathbf{c}(N, K)$ for the signless-1Stirling #s.

B4: Short answer. Show no work.

a Which is *optional*? *Writing-in-sentences.*
Writing-t-different-from-+. *Writing-LARGE.* *Um...*

b The Bell-number recurrence relation we discussed in class is

$$\forall K \in \mathbb{N}: B(K+1) = \sum_{n=\ell}^K [\mu_n \cdot B(n)], \text{ where } \ell = \dots \text{ and } \mu_n = \dots$$

[N.B: The μ_n numbers may depend on K .]

c For $N \geq K \geq 1$, we have recurrence relations:

$$\mathcal{S}(N, K) = \dots$$

$$\mathbf{c}(N, K) = \dots$$

d $\mathbf{c}(L, L-1) = \dots$ [Closed formula].

Fnc $f(n) := \mathbf{c}(n, n-2)$ is a polynomial of degree \dots .

e Define $G:[1..12] \rightarrow \mathbb{N}$ where $G(n)$ is the number of letters in the n^{th} Gregorian month. So $G(2) = 8$, since the 2nd month is “February”. The only fixed-point of G is \dots . The set of posints k where $G^{\circ k}(12) = G^{\circ k}(7)$ is \dots .

[January, February, March, April, May, June, July, August, September, October, November, December]

OYOP: In grammatical English *sentences*, write your essay on every *third* line (usually), so that I can easily write between the lines. Start each essay on a new sheet-of-paper. Please number the pages “1 of 57”, “2 of 57”... (or “1/57”, “2/57”...) I suggest you put your name on each sheet.

B5: **i** Give our formal definition of what it means for a $\pi \in \mathbb{S}_N$ to be an *even* [i.e, +1] or an *odd* permutation [i.e, -1]. This is call the *sign* of π , written, $\text{Sgn}(\pi)$.

ii Prove, $\forall \beta, \alpha \in \mathbb{S}_N$, that $\text{Sgn}(\beta \circ \alpha) = \text{Sgn}(\beta) \cdot \text{Sgn}(\alpha)$.

End of Class-B

B4: _____ 95pts

B5: _____ 55pts

Total: _____ 150pts