Differential Eqns
MAP2302

B-Makeup

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Hello. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. **Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with negative signs!)**

Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797$\cdots$.

Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$.

Write *rational numbers* as fractions: E.g $\frac{1}{2}$ and $1/3$, but not 0.5 nor 0.3333$\ldots$; **use fractions**.

B1: Show no work.

a Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. Circle one:

True! **Yes!** ***wH'at S a?sEnTENcE***

c DE $\left[\mathcal{N}(x,y)\cdot\frac{dy}{dx}\right]+\mathcal{M}(x,y)=0$ is *exact*, where

$$\mathcal{N}(x,y):=x+2y \quad \text{and} \quad \mathcal{M}(x,y):=5+y\,.$$

Its soln $y=y(x)$ satisfies $\mathbf{F}\big(x,y(x)\big)=\text{Const}$, where $\mathbf{F}(x,y)=$ ⌞.....⌟.

d DE $\left[2xy\cdot\frac{dy}{dx}\right]+[2+3x]y^2=0$ is not, alas, *exact*. Happily, multiplying both sides by (non-constant) fnc

$W(x)=$ ⌞.....⌟

gives a *new* DE which *is* exact. **Did you *Check?***

e *A* soln to $\left[f''-3f'\right](x)=-[4x+3]$ is **polynomial** $f(x)=$ ⌞.....⌟. Using parameters α and β, then, the *general* solution to $\left[h''-3h'\right](x)=-[4x+3]$ is

$h_{\alpha,\beta}(x)=$ ⌞.....⌟.

And the h with $h(0)=0$ and $h'(0)=0$ is $h(x)=$ ⌞.....⌟.

f Matrix-product $\begin{bmatrix}7&3\\8&4\end{bmatrix}\cdot\begin{bmatrix}1&3\\-2&1\end{bmatrix}=$ ⌞.....⌟.

With $\mathsf{A}:=\begin{bmatrix}7&3\\8&4\end{bmatrix}$, then the $(2,1)$-entry of A^{-1} is ⌞.....⌟.

OYOP: *In grammatical English* ***sentences****, write your essays on every* ***third*** *line* (usually)*, so that I can easily write between the lines.*

B2: Brine with $5\frac{\text{lb}}{\text{gal}}$ salt flows at rate $1\frac{\text{gal}}{\text{min}}$ *into* a tank that initially held 100gal of $2\frac{\text{lb}}{\text{gal}}$-salt brine. The tank is well-mixed, and brine is flowing *out* at rate $3\frac{\text{gal}}{\text{min}}$. So the tank will empty in ⌞.....⌟ minutes.

At time t, let $\sigma(t)$ denote the tank-salinity [in lb/gal] and use $y(t)$ for the total number of pounds of salt in the tank.

Explain how to derive a DE for $y()$. Don't just pull a DE out of the air; **explain**, using *Text* and *Pictures*, how it comes from the physical situation. Now re-write the DE in linear-DE form. Use FOLDE to solve the DE. Dividing by the amount of water in the tank at time t, gives this formula for the salinity:

$\sigma(t)=$ ⌞.....⌟ $\frac{\text{lb}}{\text{gal}}$.

As the tank approaches empty, its salinity approaches ⌞.....⌟ $\frac{\text{lb}}{\text{gal}}$. *Do Not approximate.*

End of B-Makeup

B1: ___ ___ ___ 110pts

B2: ___ ___ 75pts

Total: ___ ___ ___ 185pts

Please PRINT your name and ordinal. Ta:

Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: