Advanced Calc
MAA4102 & MAA5104

InClass-α

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Note. Fill in the blanks. You need show no work for question α-6.

α6: a Give two integers $p,q \geqslant 2$, neither prime, so that $p \perp q$: $p =$ ______ & $q =$ ______.

b Consider $x \mapsto \sqrt[3]{5x+2}$ as a map $\mathbb{R} \to \mathbb{R}$. Let φ be its inverse function. Then
$\varphi(y) =$ ______________________.

c Give a formula for a particular $f\colon (2,3] \to \mathbb{R}$ which is *continuous*, but is <u>not</u> uniformly continuous.
$f(x) :=$ ______________________.

d Define function ψ on $\mathbb{R} \setminus \{0\}$ by

$$\psi(z) \quad := \quad \sin(\tfrac{1}{z}) + 2\cos(z)\,.$$

Then
$\limsup_{z \to 0} \psi(z) =$ ______ $\liminf_{z \to 0} \psi(z) =$ ______
$\limsup_{z \to +\infty} \psi(z) =$ ______ $\liminf_{z \to +\infty} \psi(z) =$ ______

e Define $h\colon \mathbb{R} \to \mathbb{R}$ as follows: Let $h(x)$ be $x^2 - 2x + 1$, if x is irrational, and $h(x) := 4$, if x is rational. Then
$\mathrm{Cty}(h) =$ ______________________.

α7: a Give an example of sets $D,C \subset \mathbb{R}$ and a bijection $f\colon D \twoheadrightarrow C$ such that f is strictly monotone and *continuous*, and yet $g := f^{-1}$ is <u>not</u> continuous. Draw a labeled *picture* of f, using an entire sheet of paper. Prove that f is continuous.

b Produce a particular point $P \in C$ at which g is not continuous. Produce, with proof, a witness $\varepsilon > 0$ of discontinuity of g at P.

End of InClass-α