Staple!

Team A___

Sets and Logic MHF3202 1079 — **Home-A** — Prof. JLF King, Wednesday, 20Sep2023

Due **BoC, Wedn., 27Sep.**, wATMP! **Print** this problem-sheet; it is Page 1 of your write-up, **with the blanks filled in** (handwritten). Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$. [Put ordinals and Team-# and sign Honor Code.]

A1: *Show no work.*

a) $B^E = \sum_{j=0}^{59} \binom{59}{j} \cdot 4^{2j}$, for posints $B=$ ____ & $E=$ ____.

b) $\forall x,z \in \mathbb{Z}$ *with* $x < z$, $\exists y \in \mathbb{Z}$ *st.:* $x < y < z$. *T F*

$\forall x,z \in \mathbb{Q}$ *with* $x \neq z$, $\exists y \in \mathbb{R}$ *st.:* $x < y < z$. *T F*

For all sets Ω, *there exists a fnc* $f{:}\mathbb{R}\to\Omega$. *T F*

c) LBolt gives $G := \text{GCD}(413, 294) =$ ____ . And $413S + 294T = G$, where $S=$ ____ & $T=$ ____ are integers.

d) On $\Omega := [1..29]\times[1..29]$, define binary-relation $\mathbf{C}$ by: $(x,\alpha)\,\mathbf{C}\,(y,\beta)$ IFF $x\cdot\beta \equiv_{30} y\cdot\alpha$. Statement "*Relation* $\mathbf{C}$ *is an* ***equivalence relation***" is: *T F*

Carefully TYPE *your two essays, double–spaced. I suggest* LaTeX*, but other systems are ok too.*

A2: [A dodecahedron is a regular polyhedron having 12 faces, 20 vertices and 30 edges; the faces are pentagons.] Two vertices of a regular dodecahedron are ***cousins*** if they are *distinct* vertices of a common face. [Each vertex has $[3\cdot 4] - 3 = 9$ cousins.] Write $v \sim w$ to indicate that v and w are cousins. Easily, $\sim$ is symmetric, and anti-reflexive. You can check that $\sim$ is not transitive.

A ***labeling*** of a regular dodecahedron assigns, to each vertex, a *positive integer*. A labeling is ***legal*** IFF *no* pair $v \sim w$ of vertices is assigned the same label.

i) Prove there is no legal labeling with vertex-sum [the sum of the 20 labels] equaling **55**.

ii) Let $\mathbb{S} \subset \mathbb{Z}_+$ be the *set* of vertex-sums obtainable from legal-labelings. Characterize $\mathbb{S}$ explicitly, with proof. You will likely need to construct some particular legal-labelings. [You showed, above, that $\mathbb{S} \not\ni 55$.]

iii) After you solved this problem, submit a prompt to some GenerativeAI and grade the AI's response.

A3: Each dot of W many, gets one of 4 colors. The minimum W *guaranteeing* that at least 3 dots have the same color is $W =$ ____ . *Prove* your answer, and show that $W-1$ is insufficient.

With this W, the $W{\times}H$***-grid*** is $\mathbf{\Gamma} := [1..W]\times[1..H]$, for an H you will determine. A subset $\mathbf{\Sigma} \subset \mathbf{\Gamma}$ of form

$$\mathbf{\Sigma} := \{x_1, x_2, x_3\} \times \{y_1, y_2\}$$

where $x_1 < x_2 < x_3 \le W$ and $y_1 < y_2 \le H$ are positive integers, is a $3{\times}2$-***subgrid*** of $\mathbf{\Gamma}$. The minimum H *guaranteeing* that each 4-coloring of $\mathbf{\Gamma}$ admits a *monochromatic* $3{\times}2$-***subgrid*** is $H =$ ____ .

Prove that your H is sufficient. Prove that $H-1$ is *not* sufficient.

a) Now allowing C many colors, compute the corresponding values W_C and H_C.

b) Can you generalize to 3 dimensions? Further? [E.g, what is the smallest $L \in \mathbb{Z}_+$ s.t each 4-coloring of $W{\times}H{\times}L$ has a monochromatic $3{\times}2{\times}2$ subgrid?]

A1:	___ ___	80pts
A2:	___ ___ ___	130pts
A3:	___ ___	85pts
Total:	___ ___ ___	295pts

Honor Code: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* Name/Signature/Ord

Ord: ____

Ord: ____

Ord: ____