Staple!

Team A____

Sets and Logic
MHF3202 139A

Home-A

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Wedn, 21Sep2022

Due ~~Wedn., 28Sep2022~~ [UF closed by hurricane Ian] **BoC, Monday, 03Oct2022**, wATMP! **Print this problem-sheet**; it is the first page of your write-up, with the blanks filled in (handwritten). Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$. [Put ordinal and Team-# and sign HONOR CODE.]

A1: *Show no work.*

___ ___ a) $B^E = \sum_{j=0}^{59} \binom{59}{j} \cdot 4^{2j}$, for posints B= ____ & E= ____ .

___ ___ b) $\forall x,z \in \mathbb{Z}$ *with* $x < z$, $\exists y \in \mathbb{Z}$ *st.:* $x < y < z$. *T F*

$\forall x,z \in \mathbb{Q}$ *with* $x \neq z$, $\exists y \in \mathbb{R}$ *st.:* $x < y < z$. *T F*

For all sets Ω, *there exists a fnc* $f\colon\mathbb{R}\to\Omega$. *T F*

___ ___ c) The ***Threeish-numbers*** comprise $\mathcal{T} := 1 + 3\mathbb{N}$.

$\mathcal{T}$-number $385 \stackrel{\text{note}}{=} 35 \cdot 11$ is $\mathcal{T}$*-irreducible*: *T F*

Threeish $N := 85$ is **not** $\mathcal{T}$*-prime* because $\mathcal{T}$-numbers J:= ____ and K:= ____ satisfy that $N \bullet\!| [J \cdot K]$, **yet** $N \nmid J$ and $N \nmid K$.

Also, $\mathcal{T}$-GCD$(175, 70)$ = ____ .

___ ___ d) On $\Omega := [1\,..\,29]\times[1\,..\,29]$, define binary-relation **C** by: $(x,\alpha)\,\mathbf{C}\,(y,\beta)$ IFF $x\cdot\beta \equiv_{30} y\cdot\alpha$. Statement "*Relation* **C** *is an* ***equivalence relation***" is: *T F*

Carefully TYPE *your two essays, double–spaced. I suggest* LaTeX*, but other systems are ok too.*

A2: Let $\mathbf{E}_n$ be the equilateral triangle with side-length 2^n. This $\mathbf{E}_n$ can be tiled in an obvious way by 4^n many little-triangles [copies of $\mathbf{E}_0$]; see picture on blackboard. The **"*punctured* $\mathbf{E}_n$"**, written $\widetilde{\mathbf{E}_n}$, has its topmost copy of $\mathbf{E}_0$ removed.

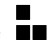
A ***(trape)zoid***, **T**, comprises three copies of $\mathbf{E}_0$ glued together in a row, rightside-up, upside-down, rightside-up. [A ***zoid-tiling*** allows all six rotations of **T**.]

i) PROVE: *For each n, board* $\widetilde{\mathbf{E}_n}$ *admits a zoid-tiling.*

ii) Let Δ_k be the equilateral triangle of sidelength k; so $\mathbf{E}_n$ is Δ_{2^n}. Triangle Δ_k comprises k^2 little-triangles.

For what values of k *does* Δ_k *admit a zoid-tiling?*

For which k *does* $\widetilde{\Delta_k}$ *admit a zoid-tiling?*

iii) An ***Lmino*** (pron. "ell-mino") comprises three  squares in an "L" shape (all four orientations are allowed).

Let $\mathbf{S}_n$ be the $2^n{\times}2^n$ square board, comprising 4^n ***squaries*** (little squares). Have $\widetilde{\mathbf{S}_n}$ be the board with one corner squarie removed. Shown in class is an inductive proof that each $\widetilde{\mathbf{S}_n}$ is Lmino-tilable (by $[4^n - 1]/3$ Lminos, of course). Further, with $\mathbf{S}'_n$ denoting $\mathbf{S}_n$ with an *arbitrary* puncture, we proved that every $\mathbf{S}'_n$ is Lmino-tilable.

Generalize this to three-dimensions. Let $\mathbf{C}_n$ denote the $2^n{\times}2^n{\times}2^n$ cube, $\widetilde{\mathbf{C}_n}$ the corner-punctured cube, and let $\mathbf{C}'_n$ be $\mathbf{C}_n$ but with an arbitrary ***cubie*** removed.

What is the 3-dimensional analog of an Lmino? Calling it a **"*3-mino*"**, how many cubies does it have? [Provide a drawing of your 3-mino.] PROVE: *Every* $\mathbf{C}'_n$ *admits a 3-mino–tiling.* [Provide also pictures showing your ideas.]

iv) Generalize to K-dim(ensional) space, with $\mathbf{C}_{n,K}$ being the $2^n{\times}\overset{K}{\cdots}{\times}2^n$ cube, having $[2^n]^K = 2^{nK}$ many K-dim'al cubies. As before, let $\mathbf{C}'_{n,K}$ be $\mathbf{C}_{n,K}$ with an arbitrary cubie removed.

What is your K***-mino*** *with which you will tile, and how many cubies does it have?* (So a 2-mino is our Lmino.) PROVE: *Every* $\mathbf{C}'_{n,K}$ *admits a* K*-mino–tiling.*

A3: For $K = 0, 1, 2, \ldots$, define sum

$$\mathcal{S}_K := \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{K\cdot[K{+}1]} \stackrel{\text{note}}{=} \sum_{n=1}^{K} \frac{1}{n\cdot[n{+}1]}.$$

Find a closed-form [no summation sign, nor dot-dot-dot] for $\mathcal{S}_K$. Prove your formula correct by induction on K.

A1:	___ ___	90pts
A2:	___ ___ ___	130pts
A3:	___ ___	75pts
Total:	___ ___ ___	295pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

.... Ord: ___

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.... Ord: ___