Staple!

Team:_____

DynSys MTG 6401 — **Home-A** — Prof. JLF King, *Touch*: 6May2016

Hello. Take-home is due by **3PM, Thursday, 15Oct2009,** slid completely **under** my office door, LIT402.

Here, we only consider bi-mpts on a probability space, e.g $(T : X, \mathcal{X}, \mu)$ or $(S : Y, \mathcal{Y}, \nu)$. All mentioned subsets of a measure-space are assumed measurable.

A1: Please prove this result.

1: First Cesàro Lemma. *Suppose that* $\vec{\mathbf{b}} := (b_n)_{n=1}^{\infty}$ *is a decreasing (non-increasing) sequence of real numbers. Then the following limit exists in* $[-\infty, \infty)$ *and this equality holds:*

$$*: \qquad \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} b_k \;=\; \inf_n b_n\,.$$

I.e $\mathbb{A}_{\infty}^{k}(b_k) = \inf_k b_k$. ◊

Aside: You may use either $\mathbb{A}_{\infty}^{k}(b_k)$ or $\mathbb{A}_{\infty}(\vec{\mathbf{b}})$ to abbrev. LhS$(*)$.

A2: Fix $B \overset{\mu}{>} 0$. For $z \in X$, define

$$\mathcal{R} = \mathcal{R}_B := \{n \in \mathbb{Z} \mid \mu(B \cap T^{-n}B) > 0\}\,;$$
$$\rho(z) = \rho_B(z) := \{n \in \mathbb{Z} \mid T^n z \in B\}\,.$$

__ __ **a** Prove that $\mathcal{R}$ has ***bounded gaps***. (The general term is "syndetic". In an abelian topological group G, a subset $\mathcal{R} \subset G$ is ***syndetic*** if *there exists* a compact $K \subset G$ st. $\mathcal{R} + K$ (the set of all sums) is the whole group.)

__ __ **b** The Birkhoff thm implies that a.e $z \in X$ hits B with a limiting frequency.[♡1] Now assume that T is ergodic. Thus for a.e z: $\mathrm{Den}(\rho(z))$ equals $\mu(B)$.

Construct an ergodic T and set $B \overset{\mu}{>} 0$ st. for a.e z in X: *The set* $\rho(z)$ *does not have bounded gaps.*

[♡1] A frequency that depends on z, or rather, on the ergodic component that z lies in.

A3: Let S be the shift on $Y := \{0,1\}^{\mathbb{Z}}$, equipped with independent $(\frac{1}{2}, \frac{1}{2})$-measure. (I.e, S is the ***Bernoulli 2-shift***.)

i Construct an $f \in \mathbb{L}^{\infty}(\nu)$ st.

$$\int_Y f = 0 \qquad \text{and} \qquad \int_Y |f| > 0\,,$$

for which the Mean Ergodic Thm conclusion *fails* in the $\mathbb{L}^{\infty}$-norm.

ii Can you construct a rank-1 trn T and $\mathbb{L}^{\infty}$-fnc f for where the same failure occurs?

[*Hint:* For both parts, LARGE, colorful, pictures may be of use.]

End of Home-A

A1:	__ __	95pts
A2:	__ __	95pts
A3:	__ __	95pts
Total:	__ __ __	285pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ....

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