Staple!

Team:____

Algebra 1 MAS4301 3175 **Home-A** Prof. JLF King Monday 03Feb2003

Hello. The order of your hand-in should be: PROBLEM SHEET (P-S, this side up), TYPESETTING CONVENTIONS (if any), followed by the write-up to each question.

General instructions are on the CHECKLIST. *Each member* of the team must retain a *complete copy* of the team's essays, **including diagrams**.

Use "$f(x)$ notation" when writing fncs; in partic., for trig and log fncs. E.g, write "$\sin(x)$" rather than $\sin x$ or $[\sin x]$. Be sure to write unambiguously, e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$.

A1: Please write a proof for #51Page85. [*Hint:* If α, β are involutions, is $\alpha\beta\alpha$ an involution? Could $\alpha\beta\alpha$ be ε?] (*Jog:* There is no gp with exactly two order-2 elements.)

A2: Please write a proof for #13Page90. (*Jog:* When is the set of N^{th} powers a gp?)

A3: Prove #32Page91. Pictures will help. (*Jog:* A cyclic gp with exactly one non-trivial proper subgp looks like what?)

A4: For each $x \in \mathbb{R}$, we have a vertical and a horizontal shear (viewed as acting on column-vectors):

$$\mathsf{H}_x := \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \qquad \text{and} \qquad \mathsf{V}_x := \begin{bmatrix} 1 & 0 \\ x & 1 \end{bmatrix} .$$

Let ***shear*** hence mean either an H_x or a V_x.

Prove that the shears generate all of $\mathrm{SL}_2\,\mathbb{R}$. What is a det=1 matrix M requiring the largest number of shears, when written as a shear-product? –and how many shears does it need?

A5: (Altered) On the 4×4 TTT (TicTacToe) board, let G denote the TTT-automorphism group. This is the set of self-bijections of $[1\,..\,4] \times [1\,..\,4]$ which preserve all TTTs. A particular TTT-auto is the ***swizzle***, S: It exchanges each corner square with the central square that it (diagonally) touches; and it does The Right Thing (tm) on the edge squares.

a Find a TTT-auto T which is **not** in the subgp $\langle \text{Isometries}, S\rangle$, yet $\langle \text{Isometries}, S, T\rangle$ is all of G .

b Compute the order of G. Find a set of *involutions* which generates G. What is a *minimum cardinality* generating set for G?

Compute the center of G; what is its order?

c For $K = 1, 2, \ldots$, let G_K denote the TTT-automorphism gp of the $K\times K$ board. Writing K as $K = 2H$ or $K = 2H+1$, as K is even or odd, compute the order of G_K in terms of H. How many *really different* (i.e, non-isomorphic) first moves are there?

What is the smallest generating set that you can find for G_K?

d On the 4×4×4 TicTacToe board (Qubic), what is a 3-dim'al analog of The Swizzle? How many *really different* first moves are there? (ExtraCredit: Give a generating set for the gp TTT-autos, G_{Qubic}.)

A1:	___ ___	50pts
A2:	___ ___	30pts
A3:	___ ___	60pts
A4:	___ ___	95pts
A5:	___ ___ ___	135pts
Total:	___ ___ ___	370pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord:

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