Topo 1: MTG4302 *and* MTG5316 | **Class-A** | Prof. JLF King 6Sep2015

"*Touching All the Bases*"

Exam date **27Oct1997**.

Abbreviations. MS, metric space. TS, topological space. For a TS Y, we use $\overline{A}$ for the closure of A, and $A^c = X \setminus A$ for the complement of A.

A1: Here Y is a set.
A family $\mathcal{T} \subset \mathcal{P}(Y)$ is a ***topology*** on Y if...
Family $\mathcal{B} \subset \mathcal{P}(Y)$ is a **"*base* of a topology"** if...
In a TS X: That sequence $x_n \to z$ means that....
TS X is ***sequentially-compact*** if ...

A2: We have a MS (X, μ). Prove or disprove: For each sequence $D_1, D_2, \ldots$ of subsets of X,

$$\overline{\bigcup_1^\infty D_n} = \bigcup_1^\infty \overline{D_n}.$$

A3: Suppose $\{(X_\alpha, \mathcal{T}_\alpha) \mid \alpha \in A\}$ is a family of topological spaces. Let

$$\Omega := \prod_{\alpha \in A} X_\alpha,$$

equipped with the product topology.

a Describe the standard pre-base, $\mathcal{P}$, for this product topology.

b Consider points $g, f_1, f_2, \dots \in \Omega$. Prove that $f_n \to g$ (w.r.t the product topology, of course) IFF

For each α: $f_n(\alpha) \to g(\alpha)$, as $n \nearrow \infty$.

A4: Suppose $\{(X_k, d_k) \mid k = 1, 2, \ldots\}$ are metric spaces, and let

$$\Omega := \prod_{k=1}^\infty X_k,$$

equipped with the product topology.

If each X_k is sequentially-compact, prove that Ω is sequentially-compact. [For the purposes of this problem, you may assume without proof that $f_n \to g$ in Ω IFF for each k, we have that $\lim_{n\to\infty} f_n(k) = g(k)$ in X_k.]

A5: Fix a set Y. A collection $\mathcal{C} \subset \mathcal{P}(Y)$ is a ***cover*** if $\bigcup(\mathcal{C}) = Y$. Say that $\mathcal{C}$ is ***locally finite***, if each $y \in Y$ is in only *finitely* many members of $\mathcal{C}$.

A cover $\mathcal{M}$ is ***minimal*** if no member can be deleted: For each $A \in \mathcal{M}$, the remainder $\mathcal{M} \setminus \{A\}$ is *not* a cover.

a Construct a cover $\mathcal{E}$ of $\mathbb{N}$ which has *no* minimal subcover.

b In contrast, suppose that $\mathcal{C}$ is a *locally finite* cover of Y. Use Zorn's Axiom to prove that $\mathcal{C}$ has a *minimal* subcover $\mathcal{M}$.

A6: Prove that an infinite metric space (M, d) always has an open set U such that both U and $M \setminus U$ are infinite.

Bonus: In the bonus query, below, YMAWOProof the following fact. *Suppose $\mathcal{B}$ is a base for the topology on a space Ω. If each $\mathcal{B}$-cover of Ω has a finite subcover, then Ω is compact.*

Bonus: Prove the following result.

1: Lemma. *Suppose X and Y are compact topological spaces. Then $X \times Y$ is compact.* ◊

End of Class-A

A1:	___ ___	35pts
A2:	___ ___	30pts
A3:	___ ___	50pts
A4:	___ ___	50pts
A5:	___ ___	50pts
A6:	___ ___	50pts
Bonus:	___ ___	20pts
Total:	___ ___ ___	265pts

Print name .... Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....