Number Theory MAS4203 8430 | **Class-A** | Prof. JLF King *Touch*: 2Jul2018

Open brain, closed book/notes. Use $\varphi()$ for the Euler phi-fnc Essays violate the CHECKLIST at *Grade Peril*!

A5: Short answer: Show no work. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

z The author of our text is Circle: **Archimedes Wiles Legendre DNE Silverman Strayer Euler Machen**

a Euler $\varphi(77000) =$ ______.
Express your answer as a product $p_1^{e_1} \cdot p_2^{e_2} \cdot \ldots$ of *primes* to posint powers, with $p_1 < p_2 < \ldots$.

b $N := \varphi(100)=$ ______. So $\varphi(N)=$ ______.
EFT says that $3^{3221} \equiv_N$ ______ $\in [0 .. N)$. Hence (by EFT) last two digits of $7^{[3^{3221}]}$ are ______.

c LBolt: $\text{Gcd}(72, 45)=$ ______ $\cdot 72 +$ ______ $\cdot 45$.
So (LBolt again) $G := \text{Gcd}(72, 45, 105)=$ ______ and ______ $\cdot 72 +$ ______ $\cdot 45 +$ ______ $\cdot 105 = G$.

d *Magic integers* $G_1=$ ______, $G_2=$ ______, $G_3=$ ______, each in $(-165 .. 165]$, are st. mapping $g: \mathbb{Z}_6 \times \mathbb{Z}_5 \times \mathbb{Z}_{11} \to \mathbb{Z}_{330}$ is a ring-isomorphism, where

$$g\big((z_1, z_2, z_3)\big) := \Big\langle z_1 G_1 + z_2 G_2 + z_3 G_3 \Big\rangle_{330}.$$

Verify for your map: $g\big((1,1,1)\big) = 1$ and $[5 \cdot 11] \bullet| G_1$ and analogously for G_2 and G_3.

e As polynomials in $\mathbb{Z}_7[x]$, let

$$B(x) := x^4 + 2x^3 + 3x^2;$$
$$C(x) := x^3 + 3x^2 + 5x + 3.$$

Write t.fol polys, using coeffs in $[-3 .. 3]$. Compute quotient and remainder polynomials,
$q(x)=$ ______ & $r(x)=$ ______,
with $B = [q \cdot C] + r$ and $\text{Deg}(r) < \text{Deg}(C)$.

f Define $G: [1 .. 12] \circlearrowleft$ where $G(n)$ is the number of letters in the n^{th} Gregorian month. So $G(2) = 8$, since the 2^{nd} month is "February". The only fixed-point of G is ______. The *set* of posints k where $G^{\circ k}(12) = G^{\circ k}(7)$ is ______.

[January, February, March, April, May, June, July, August, September, October, November, December]

g The divisor-sum $\sigma(1500) =$ ______.
Express your answer a product $p_1^{e_1} \cdot p_2^{e_2} \cdot \ldots$ of primes to posint powers, with $p_1 < p_2 < \ldots$.

A6: Please state Wilson's Thm. Now give a careful detailed proof. [Bonus for Legendre-Symbol Theorem]

End of Class-A

A-Home:	___ ___ ___	345pts
A5:	___ ___ ___	120pts
A6:	___ ___	55pts
Total:	___ ___ ___	520pts

Print name ______ Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ______