Note: I use IFF for "if and only if".

A1: SHORT ANSWER. Fill in the blanks below, expressing your answer in simplest form unless otherwise indicated. Write **DNE**, for "Does Not Exist", if the indicated operation cannot be performed.

Do not make approximations. **Show no work.** There is no partial credit for this question, so carefully verify that you have written what *you* mean. In particular, make sure that you write expressions unambiguously, e.g the expression "$1/a + b$" should be parenthesized either $(1/a) + b$ or $1/(a + b)$ so that I know your meaning. Be careful with negative signs. [Points $30 * 4 + 15 * 6 = 210$]

Let $A := \begin{bmatrix} 2 & 3 & 4 \\ -1 & -1 & 1 \\ -2 & 0 & 1 \end{bmatrix}$, $B := \begin{bmatrix} \sqrt{2} & 0 & \sqrt{2} \\ 0 & -1 & 5 \end{bmatrix}$, $C := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 13 & 14 \end{bmatrix}$, $\mathbf{u} := \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{w} := \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$.

(a1) Compute $AB =$ ____________. $BA =$ ____________. $A + 2C =$ ____________.

(a2) $\text{rref}(A) =$ ____________. and $\text{rref}(C) =$ ____________.

(a3) Circle the following vectors which are in $\text{Span}\{\mathbf{u}, \mathbf{w}\}$.

$$\mathbf{v}_1 := (2, 4, 6) \qquad \mathbf{v}_2 := (2, 4, -6) \qquad \mathbf{v}_3 := (1 + \pi, 2\pi + 2, \sqrt{\pi}) \qquad \mathbf{v}_4 := (0, 0, 0)\,.$$

(b) The number of multiplications need, in the worst case, to compute rref of an $N \times N$ matrix is about cN^k where $c =$ ______ and $k =$ ______.

Henceforth, let AT mean "Always True", AF mean "Always False" and NEI mean "NEIther always true nor always false". Below, $\mathbf{u}, \mathbf{v}, \mathbf{w}$ represent *non-zero* vectors in $\mathbb{R}^4$.

(c) If $\mathbf{w} \in \text{Span}\{\mathbf{u}, \mathbf{v}\}$ then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. AT AF NEI

(d) If $\mathbf{w} \notin \text{Span}\{\mathbf{u}, \mathbf{v}\}$ then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is lin. independent. AT AF NEI

(e) Collection $\{\mathbf{0}, \mathbf{w}\}$ is linearly independent. AT AF NEI

(f) $\text{Span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is all of $\mathbb{R}^4$. AT AF NEI

(g) $\text{Span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{u} + 2\mathbf{v} + 3\mathbf{w}\}$ is all of $\mathbb{R}^4$. AT AF NEI

(h) If none of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ is a multiple of the other vectors, then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is lin. independent. AT AF NEI

A2: [120 points] On separate sheets of paper, write out the following sentences, and complete them to give the correct definitions. Be specific with phrases "every", "some", "there exists", etc..

A collection $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_K\}$ *is* ***linearly independent*** *IFF* ...

A vector $\mathbf{w}$ *is in the* ***span*** *of* $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_K\}$ *IFF* ...

An $N \times K$ *matrix* A *is in* ***reduced row echelon form*** *IFF* ...

A3: [50 points] A system of 3 equations in unknowns $x_1, \ldots, x_5$ reduces to the augmented matrix $\begin{bmatrix} 1 & 6 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 8 & 0 \\ 0 & 0 & 0 & 1 & 5 & 0 \end{bmatrix}$. Describe the *general solution* in this form

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \alpha \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \beta \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \gamma \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \ldots$$

where each $\alpha, \beta, \ldots$ is a free variable (either x_1 or $\ldots$ or x_5), and each column vector has specific numbers in it. In particular, *how many* free variables are there?

Filename: `a-cl.LinC1998g.ams.tex`
As of: *Sunday 06Sep2015* Typeset: *6September2015*