Staple!

Ord: __ __

DynSys MTG 6401	**Class-A**	Prof. JLF King *Touch*: 6May2016

A1:	__ __ __	115pts
A2:	__ __	95pts
A3:	__ __	25pts
Total:	__ __ __	235pts

A1: Recall the Mean Ergodic Thm.

1: $\mathbb{L}^2$-MET. *Suppose* $\mathsf{U}:\mathbf{H}\to\mathbf{H}$ *is a weak-contraction,* $\|\mathsf{U}\|_{op} \leqslant 1$, *on a Hilbert space. Let* P *be orthogonal projection (an operator) onto* $\mathbf{W}$, *the (closed) subspace of* U*-invariant vectors. Let* $\mathsf{A}_N := \frac{1}{N}\sum_{k=0}^{N-1}\mathsf{U}^k$. *Then*

$$\mathsf{A}_N \xrightarrow{SoT} \mathsf{P}\,, \quad \text{as } N\nearrow\infty.$$

I.e, for each $\mathbf{v}\in\mathbf{H}$, *necessarily* $\mathsf{A}_N(\mathbf{v}) \to \mathsf{P}(\mathbf{v})$. ◊

a Prove $\mathbb{L}^2$-MET when U is a unitary operator, $\mathsf{U}^* = \mathsf{U}^{-1}$. (In this and the next part, if you want to assume that U is the Koopman operator of a mpt, that is fine.)

b Prove, for a weak-contraction U, that a vector $\mathbf{w}$ is U^*-invariant IFF $\mathbf{w}$ is U-invariant. (You may use for free that $\|\mathsf{U}^*\|_{op} = \|\mathsf{U}\|_{op}$.)

Describe the modification to get the full $\mathbb{L}^2$-MET, even if U is *not* unitary.

A2: Suppose $R = R_{\boldsymbol{\alpha}}$ is an irrational rotation on the "circle" $X := [0,1)$.

Prove that the R-forward-orbit of an arbitrary $z\in X$, is dense. (Please use $\mathcal{O}^+(z)$ for the orbit.)

A3: *Short answer; show no work.* Imagine we express points $x\in\mathbb{R}_+$ in base-4. Let $\Delta(z)\in[1\,..\,3]$ denote the *high-order **fit*** (**f**our-ary dig**it**) of the base-four numeral of z. Let $S:\mathbb{R}_+\circlearrowleft$ be the ***doubling map***, $z\mapsto 2z$. Thus

$$\frac{1}{N}\sum_{k=0}^{N-1}\Delta(S^k z) \quad \xrightarrow{N\to\infty} \quad L\,,$$

where L is the number ....

(Make sure to write the base of any logarithms you use.)

End of Class-A

Please PRINT your name *and* ordinal. *Ta:*

.... Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....