Linear Algebra MAS4105 5441 **PA-Practice** Prof. JLF King 31Aug2015

Please. Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797⋯.Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$. Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!)

PA1: Short answer: Show no work. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a) $\mathsf{M} := \begin{bmatrix} 70 & 7 \\ 1 & 2 \end{bmatrix}$. Compute M^{-1} over these three fields.

Over $\mathbb{Z}_5$: M^{-1}= ....

Over $\mathbb{Z}_7$: M^{-1}= .... Over $\mathbb{Q}$: M^{-1}= ....

b) Let $\mathsf{M} := \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ and $\mathsf{B} := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

Then M^{2005} = ... and B^{2005} = ...

c) Relative to the std basis on $\text{Poly}_{<4}$, the differentiation operator has 4×4 matrix ....

d) $\mathsf{B} := \begin{bmatrix} \sqrt{2} & 0 & -\sqrt{3} \\ 0 & -1 & 5 \end{bmatrix}$. Then B^2= ....

PA2: *Henceforth*, let *AT* mean "Always True", *AF* mean "Always False" and *Nei* mean "Neither always true nor always false". Below, $\mathbf{u}, \mathbf{v}, \mathbf{w}$ repr. *distinct, non-zero* vectors in $\mathbb{R}^4$, a $\mathbb{R}$-VS. Please circle the correct response:

y1) If $\mathbf{w} \in \text{Spn}\{\mathbf{u}, \mathbf{v}\}$ then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. *AT AF Nei*

y2) If $\mathbf{w} \notin \text{Spn}\{\mathbf{u}, \mathbf{v}\}$ then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. *AT AF Nei*

y3) Collection $\{\mathbf{0}, \mathbf{w}\}$ is linearly-indep. *AT AF Nei*

y4) $\text{Spn}\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{u} + 2\mathbf{v} + 3\mathbf{w}\}$ is all of $\mathbb{R}^4$. *AT AF Nei*

y5) If none of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ is a multiple of the other vectors, then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. *AT AF Nei*

PA2: Let $\mathsf{R} := \frac{\sqrt{2}}{2} \cdot \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$. Then

$$\mathsf{R}^{146} = \begin{bmatrix} \quad \end{bmatrix}.$$

[*Hint:* Think of the linear trn this matrix represents.]

PA3: A system of 3 equations in unknowns $x_1, \ldots, x_6$ reduces to the augmented matrix $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 6 & | -2 \\ 0 & 0 & 0 & 1 & 0 & -8 & | -3 \\ 0 & 0 & 0 & 0 & 1 & 5 & | -7 \end{bmatrix}$. Write the *general soln* in **this form**, but using the correct number of free vars:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \alpha \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \beta \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \gamma \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \delta \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \ldots$$

where each $\alpha, \beta, \ldots$ is a free variable (either x_1 or…or x_6), and each column vector has specific numbers in it. The **number** of free variables is ....

PA4: On separate sheets of paper, write out the following sentences, and complete them to give the correct definitions. Be specific with phrases "every", "some", "there exists", etc..

*The **nullspace** of* $\mathsf{T}: \mathbf{V} \to \mathbf{W}$ *is the set of all …*

A collection $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_K\}$ *is **linearly independent** IFF …*

A vector $\mathbf{w}$ *is in the **span** of* $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_K\}$ *IFF …*

An $N \times K$ *matrix* A *is in **reduced row echelon form** IFF …*

End of PA-Practice

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