

**A3:** Short answer. Show no work.Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

**a** Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth.  one:

True! Yes! **wH'at S a?sEnTENcE****A3:** \_\_\_\_\_ 70pts**A4:** \_\_\_\_\_ 35pts**A5:** \_\_\_\_\_ 35pts**Total:** \_\_\_\_\_ 140pts

**b**  $B^E = \sum_{j=0}^{59} \binom{59}{j} \cdot 4^{2j}$ , for posints  $B = \text{_____}$  &  $E = \text{_____}$ .

**c** The physics lab has atomic *zinc*, *tin*, *silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] many possibilities.

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**d** There are  $\binom{K}{J}$  many [diagonal] lattice-paths from point  $(0, 2)$  to  $(21, 7)$ , where  $K = \text{_____}$  and  $J = \text{_____}$ .

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Such a path is **bad**, if it touches the  $x$ -axis. And  $|\text{BAD}| = \binom{N}{L}$ , where  $N = \text{_____}$  and  $L = \text{_____}$ .

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OYOP: *In grammatical English sentences, write your essays on every third line (usually), so that I can easily write between the lines.* Start each essay on a new sheet-of-paper. Please number the pages "1 of 57", "2 of 57" ... (or "1/57", "2/57"...) I suggest you put your name on each sheet.

**A4:** Give a careful bijective proof of:Thm: *Fix a natnum  $N \geq 3$ . Then*

$$* \quad \llbracket N \downarrow 3 \rrbracket \cdot 2^{N-3} = \sum_{k=3}^N \llbracket k \downarrow 3 \rrbracket \cdot \binom{N}{k}.$$

**A5:** Prove, for each natnum  $N$ , that

$$\sum_{k=0}^N \left[ \binom{N}{k}^2 \right] = \binom{2N}{N}.$$