Lin.Alg. Project 3 3114

Prof. J.L. King

The *First Order Estimate* project.

Suppose we have an $N{\times}1$ vector $\mathbf{b}$ and an $N{\times}N$ non-singular matrix A, but there is uncertainty in two of the entries of A. Then there is uncertainty in each of the N entries of the vector $\mathbf{x}$ which solves the eqn.

$$A\mathbf{x} = \mathbf{b}\,.$$

Use the Method that I showed in class (using Cramer's Thm, P.196 of our text), to compute the uncertainty in each entry of $\mathbf{x}$.

Your program will take input like

$$\texttt{firstordest}\big(A, \mathbf{b},\ \big[[2,3],\ 0.01,\ [5,7],\ 0.4\big]\big)$$

This means that you've input a square non-singular matrix A. Suppose that its $[2,3]$ entry, call it y, is 5. Then this means that y is *really* 5 ± 0.01. Letting z denote A's $[5,7]$ entry, and supposing $z = -14$, then this means that z really equals -14 ± 0.4.

Your program should check that A is square, is non-singular, and that its size N is large enough (in this case, N must be at least 7) for the following entries to be in the matrix. Your program should check that each uncertainty, the numbers 0.01 and 0.4, are non-negative.

When run, your program should print:

```
In the solution to Ax=b, here is the first
order uncertainty in each of the coordinates
of x.
  x_1 equals ⟨value⟩ plus or minus ⟨error⟩.
  x_2 equals ⟨value⟩ plus or minus ⟨error⟩.
   ...
  x_N equals ⟨value⟩ plus or minus ⟨error⟩.
```

To compute the value and error associated with, say, x_2, do the following: Directly solve $A\mathbf{x} = \mathbf{b}$ (just use a Maple command). Now use Cramer's rule, as we did in class, to write

$$x_2 \;=\; \det\big(A_2(\mathbf{b})\big) \;/\; \det(A). \tag{E1}$$

Now regard both $A_2(\mathbf{b})$ and A as functions of the values in positions y and z. Compute the partial derivatives

$$\frac{\partial x_2}{\partial y} \quad\text{and}\quad \frac{\partial x_2}{\partial z}$$

(Use Maple's differentiation command, applied to the righthand side of (E1)).

Then the first-order error in x_2 is the sum

$$(0.01)\cdot\left|\frac{\partial x_2}{\partial y}\right| + (0.4)\cdot\left|\frac{\partial x_2}{\partial z}\right|.$$

Last comment. Do read your email for an update.

Filename: `P3-project.LinC1998g.ams.tex`
As of: *Wed Apr 15, 1998* Typeset: *6September2015*